A FUZZY COLOR CREDIBILITY APPROACH TO COLOR IMAGE FILTERING

Constantin Vertan, Nozha Boujemaa

IMEDIA Project
INRIA Rocquencourt, France
Constantin.Vertan@inria.fr, Nozha.Boujemaa@inria.fr

Vasile Buzuloiu

Image Processing and Analysis Laboratory (IPAL)
Politehnica University of Bucharest, Romania
buzuloiu@alpha.imag.pub.ro

ABSTRACT

This contribution proposes a fuzzy approach to color image filtering by the fuzzy modeling of the concept of color credibility. Based on the perceptual notion of color resemblance, the colors are modeled as fuzzy sets in the CIELAB color space. The filtering principle is to select at the filters output the color that is the most credible with respect to the rest of colors within the filtering window. Although the approach does not make any assumption on the desired filter type, the result is similar to a vector median-type filter.

1. INTRODUCTION

During the last decade much attention has been devoted to the study of the arising field of vector (or multichannel) image processing. The progresses of the multispectral sensing devices and the growing power of computers enabled and imposed the consideration of images characterized in each pixel by a vector (color images being the most common example). Since the beginning, it was noticed that the “stack of scalars” approach to multichannel images (and thus, the independent processing of each component according to the scalar techniques) was not the most appropriate one, since it neglected the inherent inter-channel correlation. Apart the approaches that proposed the decorrelation of signals [1], most authors deal with vector processing.

The nonlinear multichannel filtering found its way with the introduction of the Vector Median Filter (VMF) in [2]. The VMF provided a method for ordering the color vectors within the filtering window, and thus enabling the selection of the most central placed – the median. The vector ordering approaches are based on the guidelines established in [3]: since it is not natural, nor obvious to rank vectors according to something that is close to the scalar order, the choice is to reduce the mathematical relation constraints and accept the principle of sub-ordering. Classified as marginal (in fact the independent component processing), conditional, partial and reduced, the use of sub-ordering principles generated many median- and morphological-like filters. The basic VMF [2] is representative for the category of reduced ordering based filters; their principle is to map each vector into a scalar, rank the scalars and define the rank of the vectors according to the rank of their corresponding scalar. Usually the scalars are derived within a distance-based approach. The distances are either Euclidean (or other $L_j$ norm) distances to fixed points or aggregate distances (as in [2]); it was proposed in [4] to use rather the inter-vector angles in order to match better their directional characteristics (approach known as directional processing).

Either way, the ranking techniques have the purpose of selecting the best placed vector (according to a centrality criterion) within the vectors selected by the filtering window; that means the outcome of the filter is a vector from its input set. Another approach is to combine the selected vectors in a convex linear combination, the weights being adaptively determined [5].

In this paper we propose a different approach: to use a fuzzy model for the color and to measure for a color set (color vectors selected by the filtering window) the credibility of each vector with respect to the others. The output of the filter will be the most credible color vector.

2. FUZZY COLOR MODELING

Since its introduction by Zadeh [6], fuzzy logic was extensively used to model uncertainty and indetermination that naturally arise in both the spatial and the values domains. Usually the fuzzy technique label is simply used for denoting positive sub-unitary numbers used as weighting factors [5]. Other approaches use fuzzy variables in order to combine different filters, often driven by mutually exclusive criteria, into a unitary processing block [7], [8]. The approach proposed in [9] is based on the definition of a fuzzy distance and a fuzzy ordering, both based on the $\alpha$-cuts of a fuzzy set that ultimately produce a fuzzy median. The extension to vector fuzzy numbers is possible by the so-called angle-decomposition principle. Still, no explicit application for color image filtering is provided.
Since the early beginnings of color science, the problem of color representation was intensively studied. Its basis was set by Maxwell, who showed that any color can be matched by a mixture of properly weighted primary stimuli. Later referred as the trichromaticity principle and supported by physiological and anatomical studies, the representation of the colors as triples is generally used [10]. The RGB color space is the most frequently used color representation method in image processing [10], [11]. Its many limitations imposed the development of many derived classes of color representation. The basis for their definitions are the perceptual motivation, the uniform chromaticity scaling, the linear transformation of the RGB or a combination of them.

The selection of a color space is crucial to any color object related problem. The family of HSV (Hue - Saturation - Value) color spaces is the typical paradigm of perceptual-based color description, and its use in image segmentation provided significant results. The CIELAB and CIE LUV [10], [11] color spaces are used for their capability of representing color difference through Euclidean distances.

The CIELAB color space also allows to express the tolerance of the human perception to small color changes; the McAdams ellipses [10] are transformed in almost equal disks. The Just Noticeable Color Difference or JND (the distance between two almost indistinguishable colors) equals 2.3. This property of the human visual system seems to be more suited as the basis of a fuzzy development: subsequently we will consider every color $c_k$ as a fuzzy set, $C_k$. For every possible color $c$ of the color space, the membership degree to the fuzzy set $C_k$ is $\mu_{C_k}(c)$ and must represent the credibility of the color $c$ with respect to the color $c_k$, i.e. their perceptual similarity. As the colors are represented as CIELAB triples, the perceptual inter-color distance is measured by the Euclidean inter-color distance $d = \|c_k - c\|$. We can define the membership degree as (1):

$$\mu_{C_k}(c) = \begin{cases} 1, & \text{if } d \leq \text{JND} \\ \exp\left(-\frac{(d/JND-1)^2}{2a^2}\right), & \text{otherwise} \end{cases} \quad (1)$$

The options in (1) correspond to the case of colors that look-alike (placed at a distance smaller than the JND) and to a smooth (Gaussian modeled) decrease in the credibility of the color $c$ as approximation of $c_k$ if they are separated by more than JND. An alternative solution is to use a piecewise linear function (2) instead of the Gaussian function; this later approach enables an increased computational speed, and will be subsequently used in the experiments.

$$\mu_{C_k}(c) = \begin{cases} 1, & \text{if } d \leq \text{JND} \\ \max\left(1 - \frac{d}{JND} - 1, 0\right), & \text{otherwise} \end{cases} \quad (2)$$

3. IMAGE FILTERING BY FUZZY COLOR MODELING

We can now formulate the problem of color image filtering (at a given pixel) as: given a set of color vectors, find the color vector that is the most credible with respect to all others; this color will be the output of the filter at the given location. This statement can be interpreted as requiring to find the most central placed color vector (if the credibility is measured in an isotropic manner).

In order do that, we must compute the fuzzy model of the union of some colors. As all the colors are fuzzy sets, their union is another fuzzy set. The membership degree of any color within this new formed fuzzy set can be computed as a T-norm [12] (or disjunction) of its membership degrees to all the colors that form that set. Various T-norms are available [13]; if $x$ and $y$ are two fuzzy quantities, their disjunction can be measured according to the Zadeh (3), algebraic (4), Lukasiewicz (5), Hamacher (6) or Einstein (7) disjunctions.

$$\sigma(x, y) = \max(x, y), \quad (3)$$
$$\sigma(x, y) = x + y - xy, \quad (4)$$
$$\sigma(x, y) = \min(1, x + y), \quad (5)$$
$$\sigma(x, y) = \frac{x + y - 2xy}{1 - xy}, \quad (6)$$
$$\sigma(x, y) = \frac{x + y}{1 + xy}, \quad (7)$$

Thus let denote by $\{c_1, c_2, \ldots, c_N\}$ the $N$ color vectors selected by the filtering window. For each color vector $c_k$, $k = 1 \ldots N$ we compute its credibility $s_k$ with respect to the set of remaining color vectors $C_k$ as:

$$s_k = \sigma(\mu_{C_k}(c_k)), \quad (8)$$

or, equivalently

$$s_k = \sigma(\mu_{C_k}(c_1), \sigma(\mu_{C_k}(c_2), \sigma(\mu_{C_k}(c_3), \ldots))) \quad (9)$$

Finally, the output of the filter $y$ is the most credible color within the input set:

$$y = c_j, \text{ with } j = \arg\max\{s_i\}, i = 1 \ldots N. \quad (10)$$

The problem that we encounter with this model is that of unrealistic membership degrees for the union of fuzzy sets: the membership of any element into the union of fuzzy sets is higher than the membership of the given element in any of the fuzzy sets (of course, this is one of the basic axioms of the fuzzy logic, but for out purposes we would like to have a membership degree in the union of sets that does not exceed the membership in the individual fuzzy sets). The maximal membership of an element within the union of fuzzy sets...
cannot be greater than the sum of its memberships within the individual fuzzy sets (11). Moreover, $\sigma(x, y) \leq 1$ as any fuzzy membership degree.

$$\max(x, y) \leq \sigma(x, y) \leq x + y. \quad (11)$$

That means that we must either work with the Zadeh T-conorm (3) or accept that the membership degrees within the union of fuzzy sets must not be limited to 1 (it sounds like a blasphemy, but we are talking of an overall color credibility, which does not necessarily need to be normalized). That is, we may introduce a pseudo T-conorm for which all the basic properties required for a disjunction [12] are true, except that this is not limited to 1; such one is the simple addition.

$$\tilde{\sigma}(x, y) = x + y \quad (12)$$

Thus, the credibility values (8) associated to each color vector within the filtering window become:

$$s_k = \sum_{i=1}^{N} \mu_{C_i}(c_k) \quad (13)$$

From the relations (10) and (13) we obtain a decision rule similar to that of the classical VMF: the vector median is the vector that has an extreme aggregate distance with respect to the other vectors. In that case we replaced just the word distance by the word credibility.

### 4. EXPERIMENTS

Although the initial filter design was not tailored to a specific kind of noise, its strong similarity with the VMF makes it suitable for the removal of long-tailed noises, such as the impulsive noise. Figure 1 presents typical filtering results of the proposed method (by the algebraic T-conorm (4) and by the addition pseudo T-conorm (12)) of an impulsive noise (channel independent) degraded image. The claim is that the visual, subjective quality assessment of the filtered images is highly satisfactory.

In Table 1 are presented some objective quality measures (SNR - Signal to Noise Ratio, MCRE - Mean Chromaticity Error, NCD - Normalized Color Difference) for the filtered images. As the measurements show, our fuzzy approach is at the same performance level as the classical (and very effective) VMF filters performed in the original RGB color space or in the CIELAB color space: the difference is due to the accepted amount of imprecision allowed by the fuzzy modeling. Still, we cannot expect that a technique that exploits the imprecision could perform better in terms of objective quantitative measurements.

Table 2 presents a comparison of the filtering performance when using different fuzzy T-conorms for the aggregation of the color resemblance degrees. As it can be easily seen, the best performance is obtained by the non-constrained pseudo fuzzy conorm (the addition).

The parameter $\beta$ controls the color fuzzy model (2); if $\beta$ is set at a small value, the support of the fuzzy set becomes smaller and fewer colors are considered similar to the color $e$. An increased color tolerance is obtained by letting $\beta$ to increase; Figure 2 presents the variation of the SNR of the filtering performance by the proposed method of impulsive noise degraded images with respect to the variation of the control parameter. It can be noticed that the data plots present a very small maximum at $\beta \approx 50$ before very slowly decreasing to an asymptotically constant value.
Table 2. Quality measures for the fuzzy filtering by different T-conorms

<table>
<thead>
<tr>
<th>T-conorm</th>
<th>SNR [dB]</th>
<th>MCRE %</th>
<th>NCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lukasiewicz (5)</td>
<td>14.36</td>
<td>0.763</td>
<td>7.146</td>
</tr>
<tr>
<td>Zadeh (3)</td>
<td>19.46</td>
<td>0.278</td>
<td>3.187</td>
</tr>
<tr>
<td>Einstein (7)</td>
<td>21.10</td>
<td>0.130</td>
<td>1.398</td>
</tr>
<tr>
<td>Hamacher (6)</td>
<td>21.35</td>
<td>0.146</td>
<td>1.678</td>
</tr>
<tr>
<td>algebraic (4)</td>
<td>22.05</td>
<td>0.096</td>
<td>1.186</td>
</tr>
<tr>
<td>addition (12)</td>
<td>24.82</td>
<td>0.069</td>
<td>0.073</td>
</tr>
</tbody>
</table>

Fig. 2. The variation of the SNR of the filtered image with respect to the modification of the \( \beta \) parameter of the color fuzzy model, for the "lena" and "peppers", impulsive noise degraded images.

5. CONCLUSIONS

This contribution proposed a fuzzy approach to color image filtering by the modeling of colors as fuzzy sets in the CIELAB color space. The filtering principle is to select at the filters output the color that is the most credible with respect to the rest of colors within the filtering window. The overall credibility is obtained by fuzzy aggregating (by T-conorms) individual inter-color resemblance degrees; we show however, that the best results are obtained by a pseudo T-conorm (addition). Although the approach does not make any assumption on the desired filter type, the result is similar to a vector median-type filter, opening thus an interesting perspective.

6. REFERENCES


