A CLUSTERING APPROACH TO VECTOR MATHEMATICAL MORPHOLOGY

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ABSTRACT

The processing and analysis of vector valued signals has become in the last decade a major field of interest. The direct extension of classical processing methods for the scalar signals is not always possible. The mathematical morphology viewed as a processing technique for gray images is such a one that cannot be extended easily. In this paper we present an approach to the vector mathematical morphology based on clustering techniques in the signal sample space.

1. INTRODUCTION

Signal processing evolved from analysing time signals to the study of 2-D signals (images) and of the multidimensional ones. In the last decades a growing interest has been devoted to the emerging vector valued (or multichannel, multispectral, multicomponent) multidimensional signals. Such signals are typically (but not restricted to) color images, where every pixel (image sample) is represented by a three component vector. The components are (generally) the amounts of pure red, green and blue that compose the local color. Other common used vector valued signals are the remote sensing images, the thermal detection images, the stereo and quadraphonic sound samples (represented by vectors having 2 to 7 components). For all these signals, it is natural, although not obvious, that the vector components are correlated. This is why the separate component processing, using the standard scalar techniques, is not appropriate.

2. MULTICHANNEL PROCESSING APPROACHES

The linear processing approach uses the same integral transforms techniques as in the scalar case, in conjunction with common decorrelation procedures, applied for the separation of individual channels. Typically, for the decorrelation, the Karhunen-Loeve and the Discrete Cosine transform are used [1], [2]. The decorrelated components are then processed separately.

A significant part of the nonlinear processing techniques are based on order statistics filters. Regardless of the particular filter type, a common processing step is the ordering (sorting) in ascending order of the signal samples within the filter window. Although the scalar ordering is obvious and very simple, there is no way to extend it to vector valued signals for obtaining a mathematically correct (according to Definition 1) and topology preserving ordering relation.

Definition 1: An ordering relation \( (x < y) \) must be reflexive \((x < x)\), transitive \((x < y \text{ and } y < z \implies x < z)\) and antisymmetric \((x < y \text{ and } y < x \implies x = y)\).

An ordering relation in a vector space that satisfies Definition 1 is the lexicographic ordering, expressed as stated in Definition 2.

Definition 2: The lexicographic ordering relation is expressed by:

\[
(\mathbf{x} <_\mathbf{Y} \mathbf{y}) \quad \exists k \in [1,n] \text{ such that } x_k < y_k, \quad \forall i = 1,k-1 \text{ and } x_i = y_i
\]

Two problems arise when applying it: the space topology is not preserved, and an importance criterion must be found for deciding the order of the components.

Barnett [3] has investigated the possible types of multidimensional orderings and retained four variants: the marginal ordering, that is equivalent to the separate ordering of each component; the partial ordering, which uses convex hull like sets; the conditional ordering, which is the scalar ordering of a single component; the reduced ordering, that performs the ordering of vectors according to some scalars, computed from the components of each vector. The reduced ordering offers the best results, being intensively used [4], [5], [6]. The computed scalar is either the generalized distance from each data sample to some fixed reference point (the average, the marginal median) [3], either a sum of distances from each data sample to some fixed characteristic points [5], either the sum of intersamples distances
These computed scalar values allow the total vector ordering (the first two situations) or just the determination of the vector median of the samples in the third case. Two main difficulties are connected to this approach: choosing of appropriate fixed points (or the local adaptation of the computed scalar [5]) according to the particular type of degradation, and the color preservation as a measure of quality. The color preservation implies that the filter output is a sample from the set of input values (that means that the average of values will not preserve colors). The adaptation of the filter is a difficult task that cannot be accomplished without heuristics and intensive computation.

3. MATHEMATICAL MORPHOLOGY OVERVIEW

The theoretical foundations of mathematical morphology have been constructed by Serra [7], extending some set operators, known as the Minkowski [set] addition and subtraction. Primarily used as a processing and analysis tool for binary images (easily interpreted as sets), the mathematical morphology extended to multidimensional scalar valued signals by some algebraic reformulations of the basic definitions. The fundamental operators, called dilation and erosion are algebraically defined as stated in Definition 3.

Definition 3: The dilation and the erosion are operators over a complete lattice, that commute with sup and inf respectively. According to this definition, the condition to be fulfilled for the existence of the morphological operators is the possibility of defining an ordering relation in the signal value space. For scalar signals the problem is trivial, but for vector valued signals we encounter the problems mentioned above. The use of some reduced ordering can be addressed [8], but the results show no dramatical improvement compared to the separate component processing (marginal ordering) and still the exact conditions (complete lattice induced by an ordering relation) are not accomplished.

In what follows we will describe how another (more intuitive) approach to the definition of mathematical morphology operators is possible, by unsupervised clustering in the signal value space. This theory will be applied to color images (although there exists no limitation regarding the number of vector components and the signal type).

4. MORPHOLOGICAL OPERATIONS BY CLUSTERING

The idea behind this approach is a very simple one: the selected pixel values (vector valued data) have to be partitioned in some classes, corresponding to their position in the sample space. Typically we will use three classes: a central class, corresponding to the ordinary pixels, and two extremum classes that will gather the outlier pixels, generated by the noise. The clustering is performed by classical algorithms, such as Basic Isodata [9], using a least mean square error (LMSE) or cluster compactness optimality criterion.

Each class may be characterized by some prototype (or centroid) that can be computed from the condition of optimality. We will define the result of the pseudo-dilation as the class centroid that lies closest to the marginal maximum and the pseudo-erosion as the class centroid that lies closest to the marginal minimum. We may also notice that a median-like-filtering can be performed on the basis of the prototype of the remaining class (the “central” one).

It is obvious that the class centroid is an “artificial” value, since there is a very small probability of having it as an input pixel value. If the creation of new colors in the resulting image is not desired, the class centroid can be approximated by its nearest data sample. The use of these two variants produces significantly different results: since the centroid is somehow equivalent to a mean value, its effect will be an increased smoothing (shown by smaller values of the NMSE (Definition 4)) and a slightly bigger color difference (shown by an increased MCRE, (Definition 5)) than the nearest neighbour of the centroid.

Any unsupervised clustering algorithm may be used. Typical examples are the Basic Isodata (k-means or LBG) [9] and the family of hierarchical clustering algorithms (the Pairwise Nearest Neighbour (PNN) [10] and the Modified Pairwise Nearest Neighbour (MPNN) [11]). The Basic Isodata is relatively well known and produces good and stable results. Its major disadvantage is the undetermined running time, given by the number of iterations through the data set until stability and the big memory requirements (all vectors must be memorized, since the clustering is performed on the entire set). The hierarchical clustering methods require a constant running time, since the vectors are processed in a single path. The principle is the following one: a bigger number of classes than desired is constructed and then the classes are successively reduced by merging. The PNN starts with an initial number of classes equal to the number of vectors to be clustered; at each processing step, the closest two classes (in terms of the reduction of the partitioning mean square error) are merged. That
Table 1: Image quality measures

<table>
<thead>
<tr>
<th>Noise type</th>
<th>NMSE %</th>
<th>MCRE %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise degraded image</td>
<td>31.048</td>
<td>4.072</td>
</tr>
<tr>
<td>NMSE</td>
<td>29.281</td>
<td>19.298</td>
</tr>
</tbody>
</table>

5. EXPERIMENTAL RESULTS

The tests were conducted on true color images (24 bits per pixel, or 8 bit for coding each color component); two classic images were used: “fish” and “waterfall”, each having 256 by 256 pixels. On each image different types of noise were superimposed: impulsive (salt&pepper) noise, 4% of pixels from each color component affected, with no interchannel correlation; normal (gaussian) noise with zero mean and variance 25 and a mixture of 2% impulsive and zero mean, variance 25 normal noise. The image quality is measured by two criteria, defined below.

Definition 4: The Normalized Mean Square Error (NMSE) is defined as

$$ NMSE = \frac{\sum_{i=1}^{H} \sum_{j=1}^{W} |\tilde{f}(i, j) - f(i, j)|^2}{\sum_{i=1}^{H} \sum_{j=1}^{W} f(i, j)^2} $$

Definition 5: The Mean Chromaticity Error (MCRE) is defined as

$$ MCRE = \frac{\sum_{i=1}^{H} \sum_{j=1}^{W} (\tilde{f}(i, j) - f(i, j))^2}{HW} $$

In the definitions above, $H$ and $W$ are the image dimensions in pixels; $\| \cdot \|$ is the $L^2$ norm and $\| \cdot \|_1$ is the $L^1$ norm, $f(i, j)$ and $\tilde{f}(i, j)$ are the original and computed values of the pixel at location $(i,j)$.

On each image a morphological filtering by opening (erosion followed by dilation) was performed; the results were compared for the marginal ordering induced morphology, the reduced ordering induced morphology by weighted Euclidean distances to marginal extremes [8], the Vector Median Filter [4] and the pseudo morphologic operators defined by clustering (with Basic Isodata and MPNN algorithms). The quality measures for the “waterfall” image are presented in Table 1; for the “fish” images the results are of the same type. Some images are presented in Figures 1 to 2 (image luminance).
6. CONCLUSIONS

In this paper we presented a novel approach to mathematical morphology (and general filtering) of vector valued multidimensional signals. The used examples were color images (for the sake of simplicity and nice representation), but the method is valid for any type of signal. The cluster approach seems to produce good results in the cases of noise elimination: the impulsive noise is totally removed (like the morphological filters usually do) but a better behaviour was noticed in the presence of normal noise. This normal noise filtering appears from computation of the classes prototypes by average type relations.

The novel MPNN clustering algorithm was found appropriate for this task: a slight migration of the classes prototypes (compared with the prototypes computed by Basic Isodata) to the marginal extremes was noticed. This fact (not desired for a general clustering problem) is welcomed for the implementation of the pseudo morphological operators. The computing time is similar to some standard multichannel filtering methods (such as the Vector Median Filter) and the required memory is very small (since the MPNN does not need the entire data set, but just a one by one evaluation of the vectors). A hardware implementation or a VLSI integration of the MPNN clustering algorithm and subsequent filtering seems at hand and this problem deserves to be addressed in the future.

7. REFERENCES