PARAMETRI DE FORMA

(bazati pe regiune)

C. VERTAN

Parametri de forma

Asociaza unei forme (multime binara in planul 2D) un set de numere prin care aceasta poate fi recunoscuta, indiferent de pozitie, dimensiune, orientare.

forme simplificate

functii

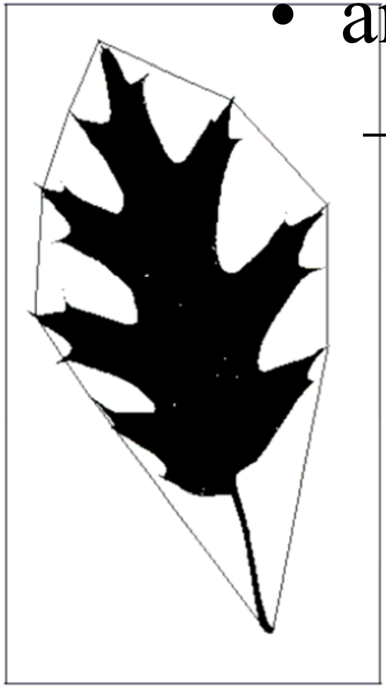
scalari

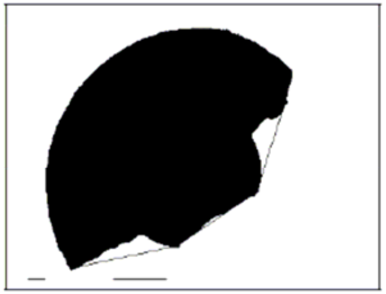
PARAMETRI DE FORMA:

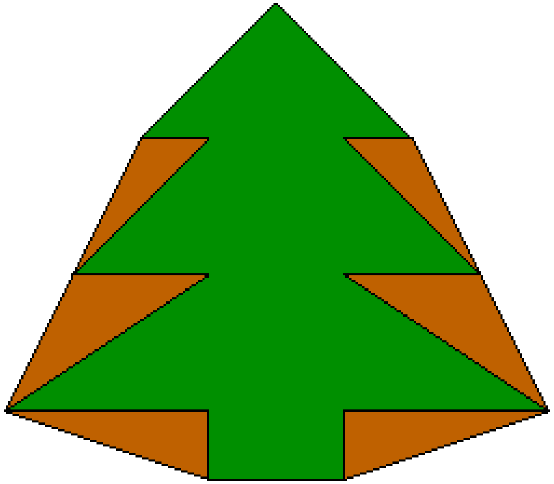
Aproximari ale formei:
Anvelopa convexa

Anvelopa convexa

- regiune convexa:
  - pt. orice $x_1, x_2 \in R$, segmentul $[\ x_1\ x_2\ ]$ este in R
- anvelopa convexa (convex hull) CH(R)
  - cea mai mica multime convexa ce contine R

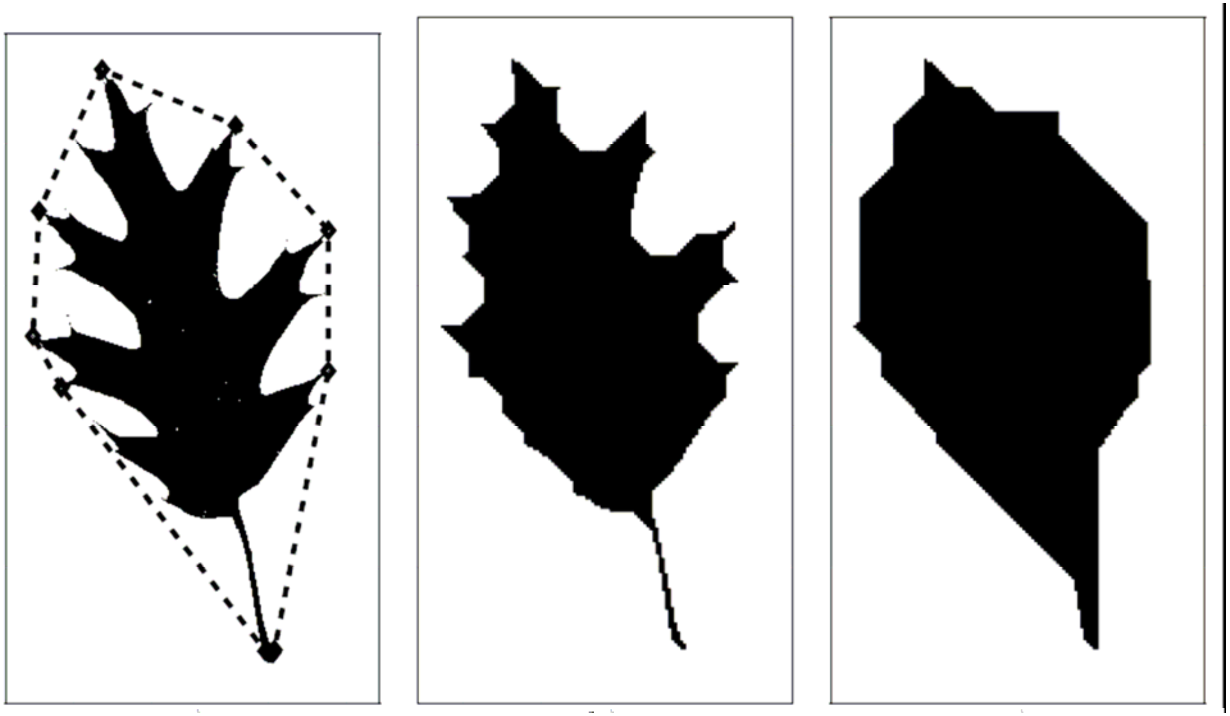






Anvelopa convexa

$$CH(A) = \lim_{n \to \infty} \left(A \bullet B(n) \right).$$



PARAMETRI DE FORMA:

Aproximari morfologice ale formei: Skeletonul morfologic

C. VERTAN

Nume echivalent : MAT - Median Axis Transform

- modelul focului in preerie

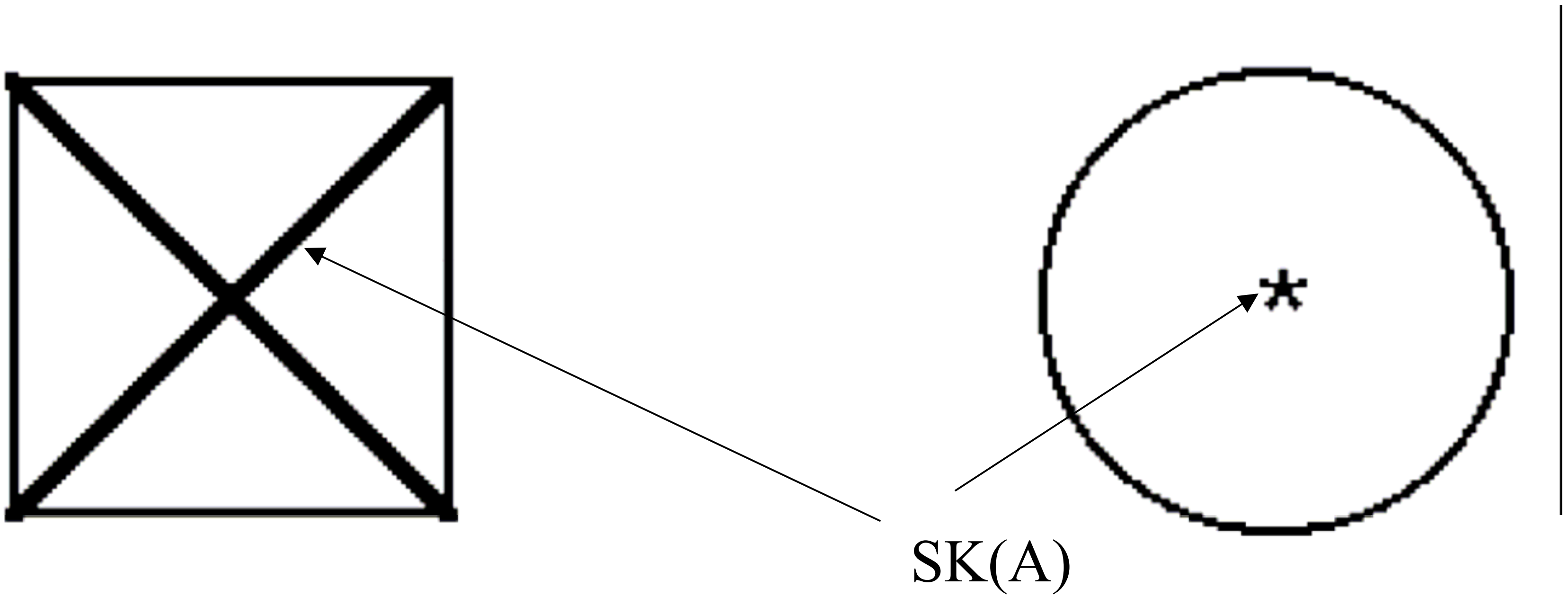
Se defineste pe baza conceptului de **disc maximal** intr-o forma A

$B_{\mathbf{x}}(r)$ | disc de centru **x** si raza *r*

$$B_{\mathbf{x}}(r) \subseteq A$$

$$B_{\mathbf{x}}(r) \subseteq B_{\mathbf{x}'}(r') \subseteq A \Leftrightarrow \begin{cases} r = r' \\ \mathbf{x} = \mathbf{x}' \end{cases}$$

Skeletonul unei forme este multimea centrelor discurilor maximal in forma.



Cum se implementeaza in cazul discret, cu operatori morfologici ?

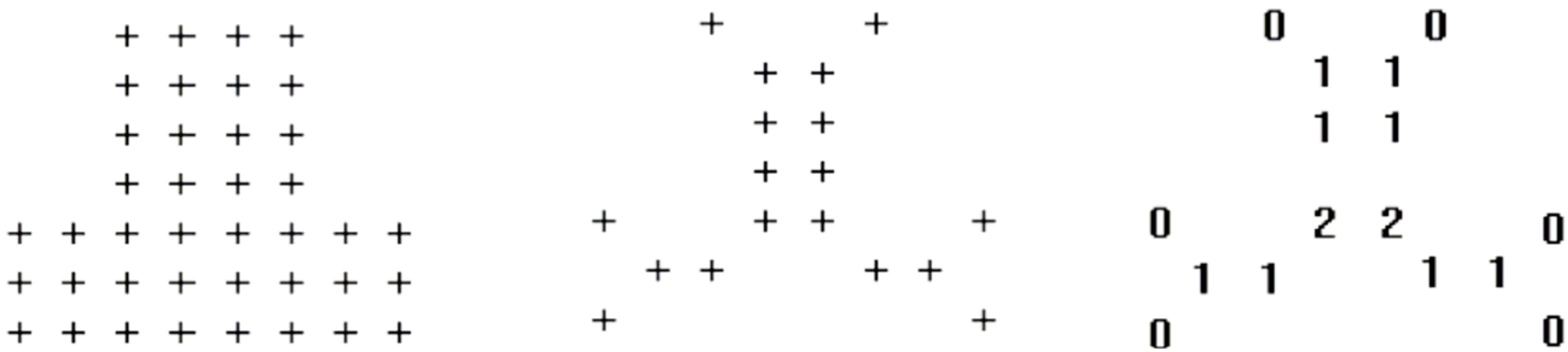
Skeletonul morfologic

$$SK(A) = \bigcup_{n=0}^{N_{\max}} S_n(A)$$

$$S_n(A) = (A \ominus nB) - (A \ominus nB) \circ B$$

$$nB = B \oplus ... \oplus B \text{ (de } n \text{ ori)}$$

B este elementul structurant ales (imagine a discului unitar)



A SK(A) SK(A)

Skeletonul morfologic : reconstructia

$$A = \bigcup_{n=0}^{N_{\max}} S_n(A) \oplus nB$$

Skeletonul morfologic : aproximarea formei

$$\widetilde{A}_k = \bigcup_{n=k}^{N_{\max}} S_n(A) \oplus nB$$

compresie/ reconstructie multirezolutie

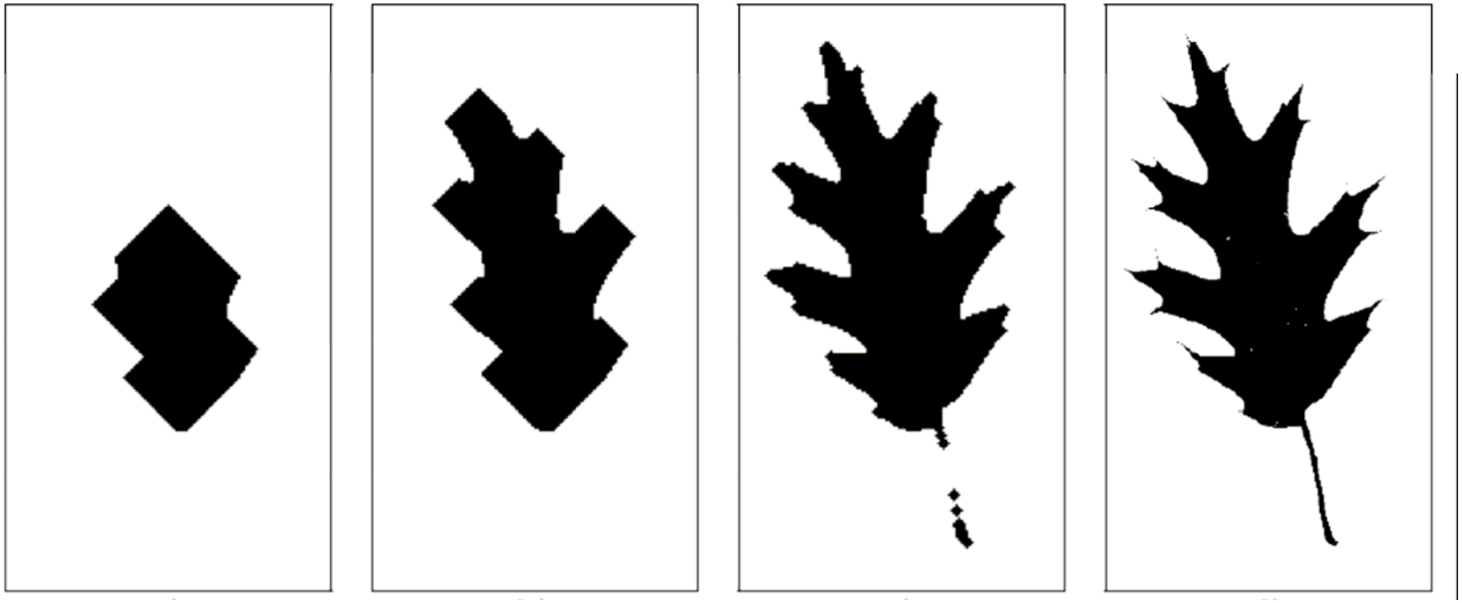


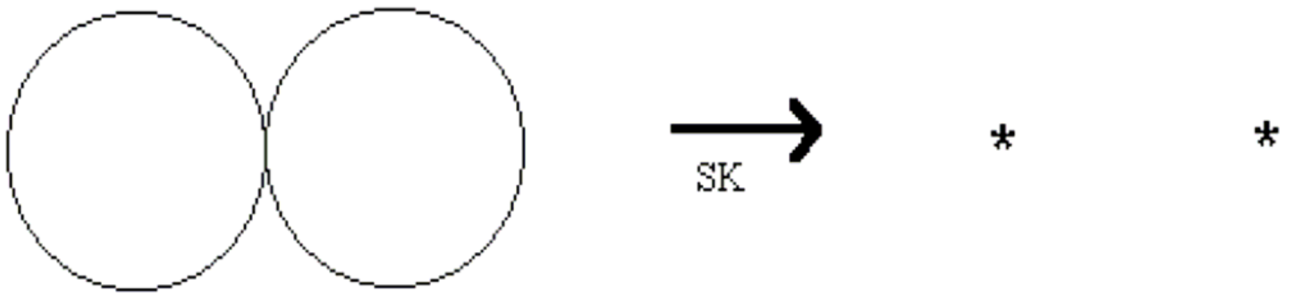
Figura 8.6: *Exemplu de reconstrucţii parţiale (aproximări) ale unei forme din skeletonul morfologic reprezentat în figura 8.5 c): reconstrucţia de ordin: a) 20, b) 10, c) 1 d) 0 (reconstrucţie perfectă a formei).*

Skeletonul morfologic : alte proprietati

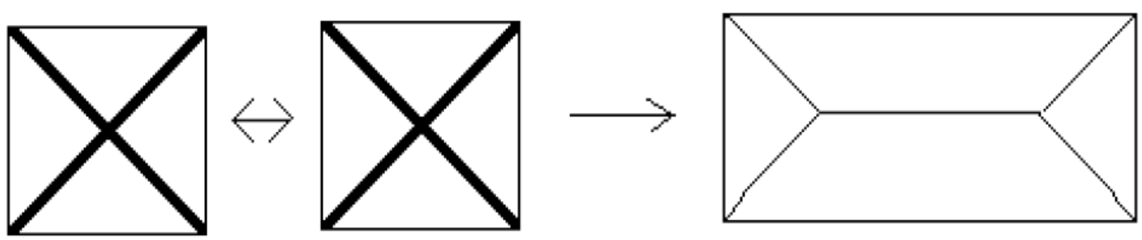
$$SK(SK(A)) = SK(A)$$ (idempotenta)

$$S_i(A) \cap S_j(A) = \emptyset \quad \forall i \neq j$$

nu pastreaza conexitatea



nu comuta cu reuniunea



slaba rezistenta la zgomot

PARAMETRI DE FORMA:

Functii caracteristice ale formei: Granulometrii

Curba granulometrica

Functie reala bazata pe masurarea prin arie a rezultatelor aplicarii unor transformari morfologice convenabil alese.

Etape:

generarea granulometriei
masurarea prin arie a elementelor granulometriei
normalizare.

Granulometrie

Set de multimi (forme plane) obtinute prin modificarea formei

de baza dupa o familie de transformari morfologice

Granulometria asociata unei forme *A* este secventa de multimi $(\Phi_A(\lambda))$¸$\lambda \in R+$, in care $\Phi_A(\lambda)$ este rezultatul prelucrarii formei *A* cu transformarea morfologica aleasa de element structurant de dimensiune λ.

Transformarea este:

crescatoare: $A_1 \subseteq A_2$, $\Phi_{A1}(\lambda) \subseteq \Phi_{A2}(\lambda)$

anti-extensiva: $\Phi_A(\lambda) \subseteq A$

verifica: $\Phi_{\Phi_{A(\mu)}}(\lambda) = \Phi_{\Phi_{A(\lambda)}}(\mu) = \Phi_A(\max(\lambda,\mu))$

Granulometrie

Ultima proprietate cere ca daca $\lambda = \mu$, sa avem idempotenta

$$\Phi_{\Phi A(\lambda)}(\lambda) = \Phi_A(\lambda)$$

Cea mai simpla transformare morfologica care indeplineste conditiile minime de a fi crescatoare, anti-extensiva, idempotenta este:

deschiderea morfologica

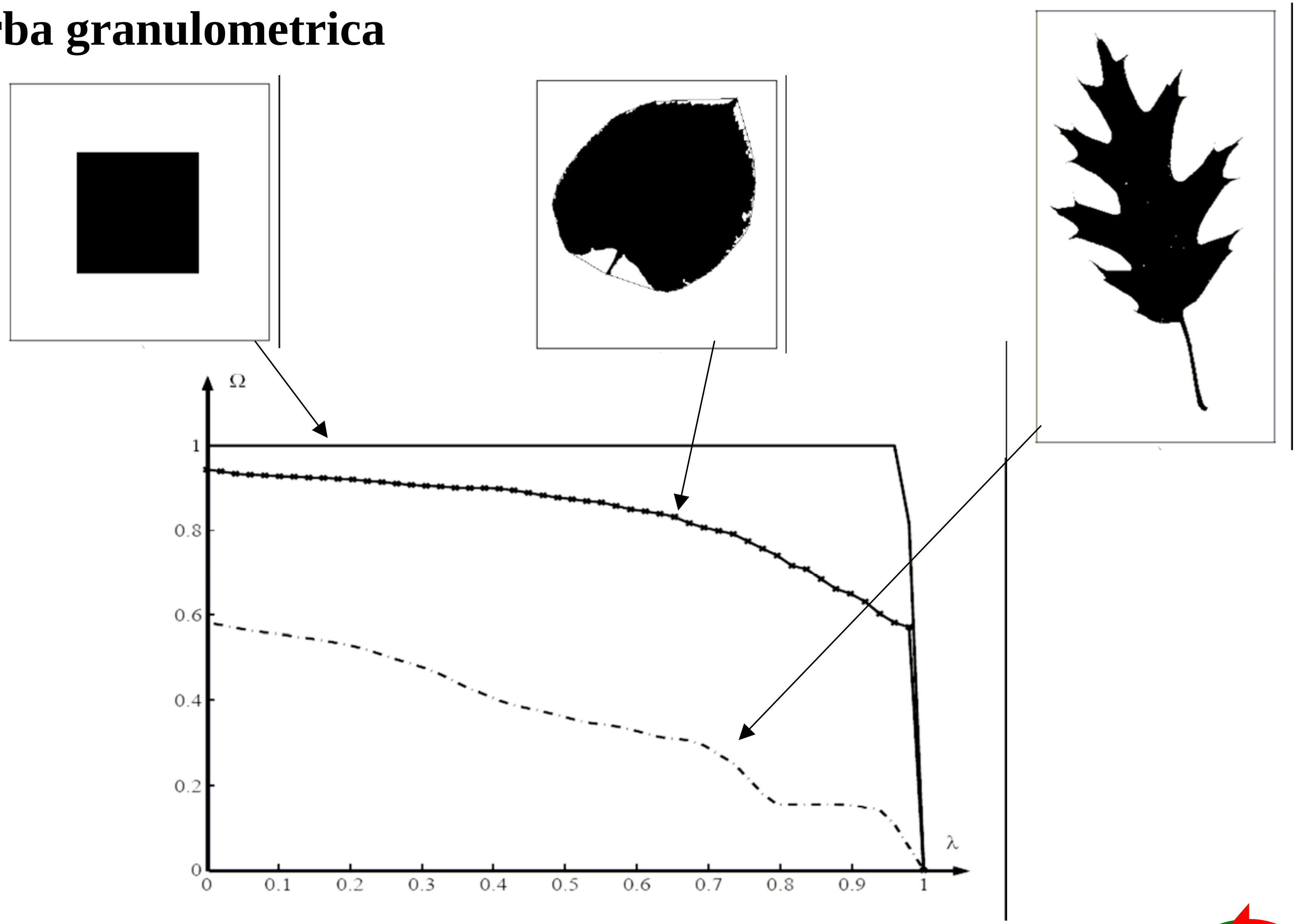
Curba granulometrica

$$\Omega_A : \mathbb{R}^+ \rightarrow [0,1]$$

$$\Omega_A(\lambda) = \frac{Aria(\Phi_A(\lambda))}{Aria\left(\lim_{n\to\infty}(A \bullet nB)\right)}$$

$$\Phi_A(\lambda) = A \bigcirc \lambda B = (A \ominus \lambda B) \oplus \lambda B^S$$

Curba granulometrica



C. VERTAN

PARAMETRI DE FORMA:

Scalari

Topologie

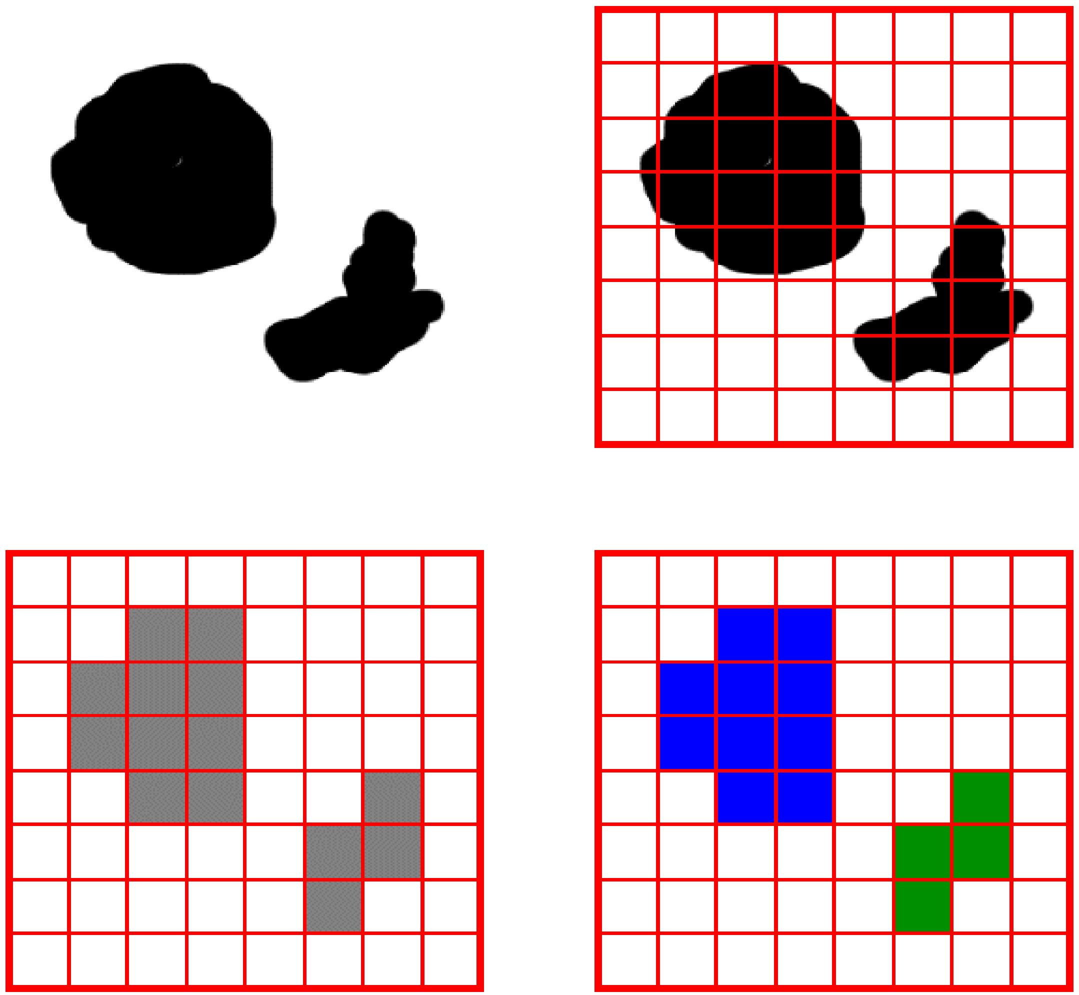
- numarul lui Euler

$$E=C-H$$

  - C – numar de componente conexe
  - H – numar de gauri

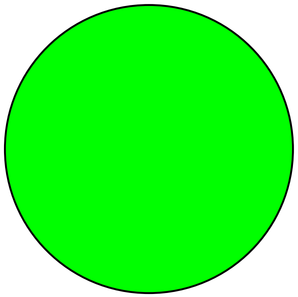
Aria = numar de pixeli

Perimetru = numar de pixeli pe contur

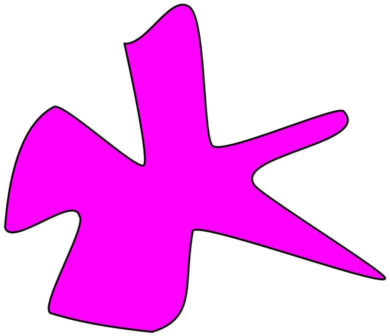


Compacitate

- P^2/A: perimetru x perimetru / arie
  - adimensional
  - minimal pentru disc
  - invariant la rotatie



compact



non compact

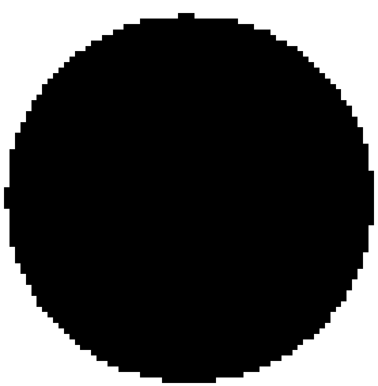
P^2/A

perimetru x perimetru / arie

normalizare: $$\frac{P^2}{4\pi A}$$

arie = 10538, perimetru = 798

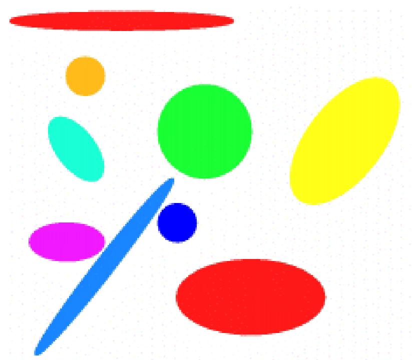
$P^2/A=60.43$, $P^2/A_{norm}=4.81$

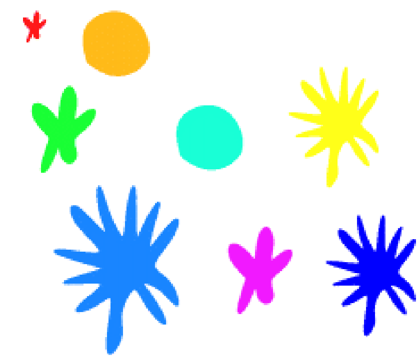


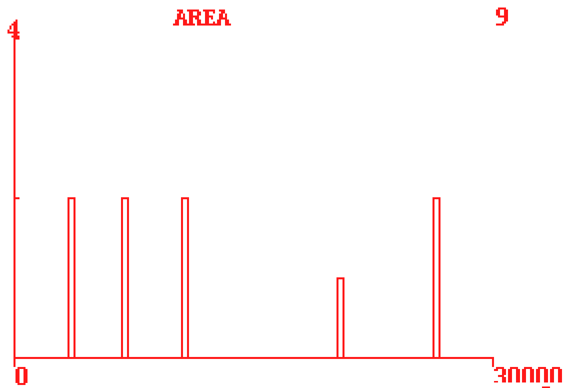
arie = 3591, perimetru = 221

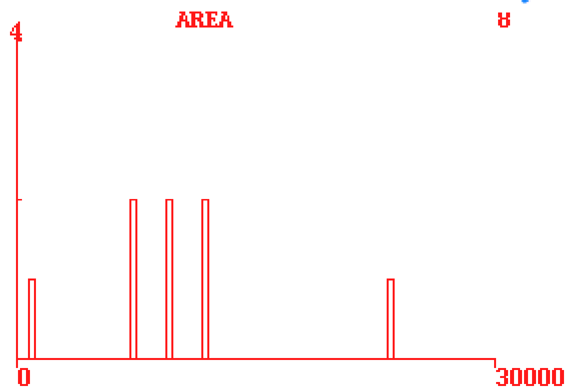
$P^2/A=13.60$, $P^2/A_{norm}=1.08$

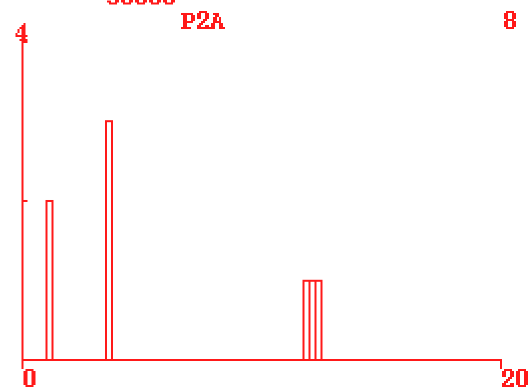
Exemple de masuratori P^2/A





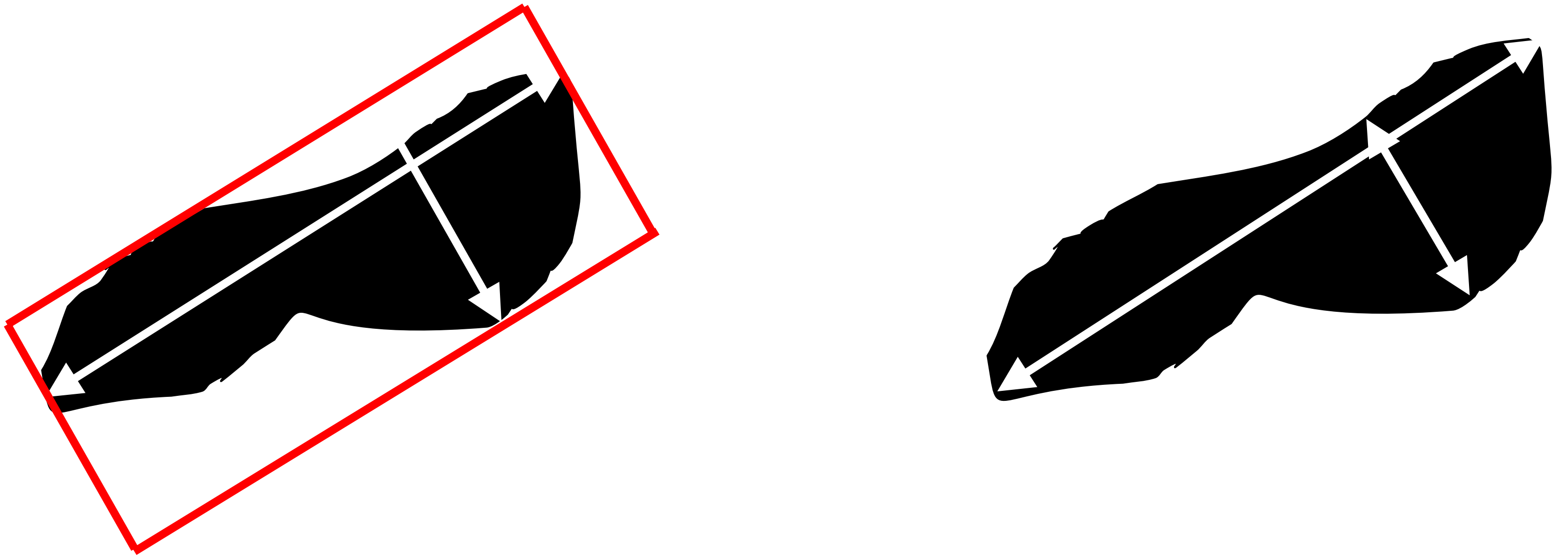






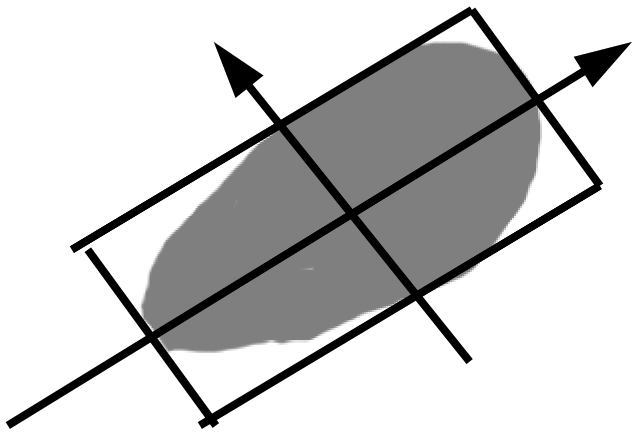
Excentricitate

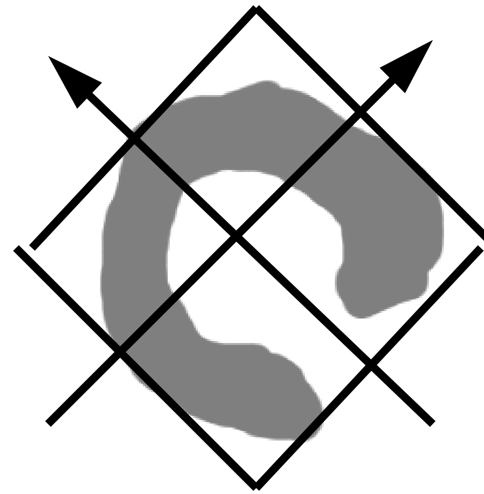
- cea mai lunga coarda/ coarda perpendiculara



Elongatie

1. raport dimensiuni pt dreptunghiul minim de incadrare

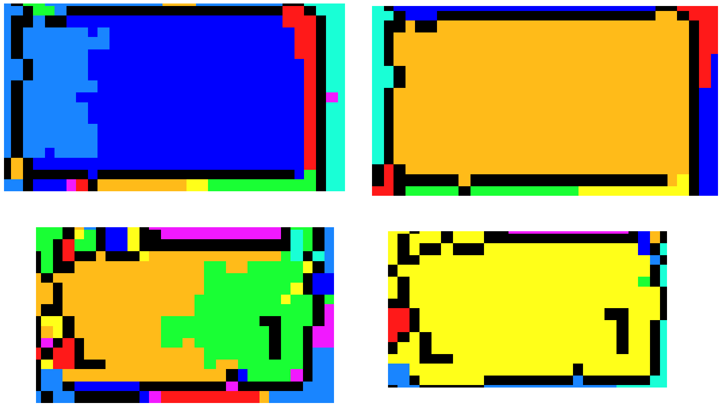


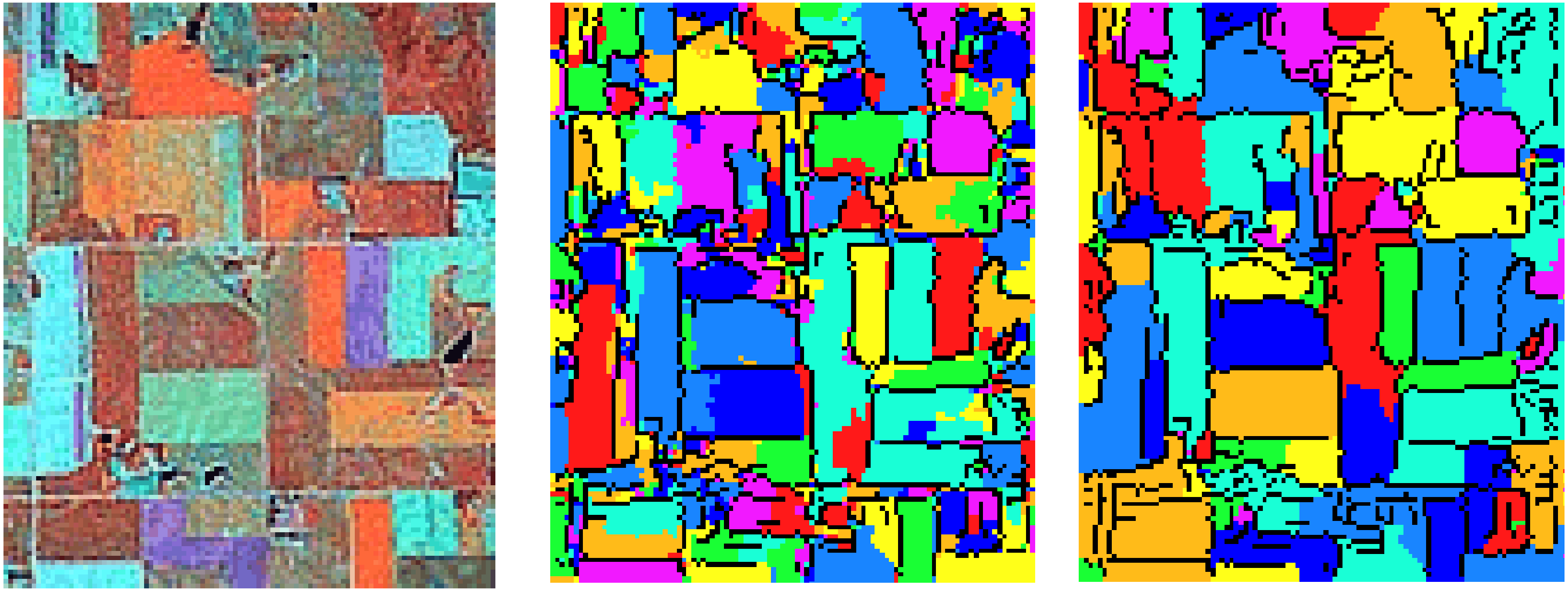


2. arie/($2d^2$)
   – d e latimea maxima
3. calea maxima

Rectangularitate

- aria regiunii/ aria dreptunghiului de incadrare



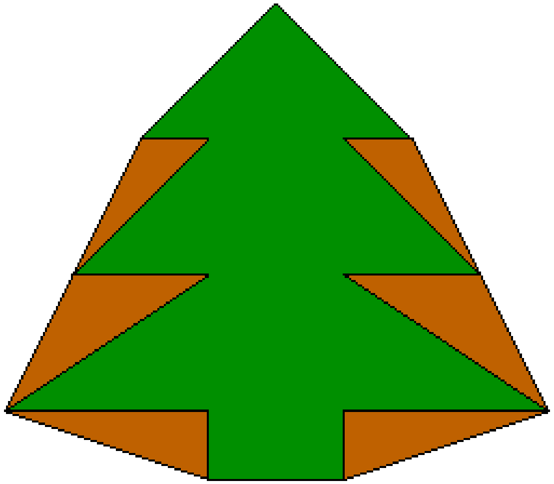


Circularitate

- raza cerc inscris / raza cerc circumscris

Convexitate

- aria / aria anvelopei convexe



PARAMETRI DE FORMA:

MOMENTE STATISTICE SI INVARIANTI

C. VERTAN

Momentele statistice ale unei forme descrise de functia binara *f*.

Forma este echivalenta cu suportul functiei (domeniul in care aceasta ia valori nenule), pe care valorile functiei sunt unitare.

$$m_{pq} = \iint_{Supp(f)} f(x,y) x^p y^q dxdy$$ coordonate continue

$$m_{pq} = \sum_{f(x,y)\neq 0}\sum x^p y^q$$ coordonate discrete

$p, q = 0, 1, 2, \ldots$

Aria

Moment statistic particular: m_{00}

$$m_{00} = \iint_{Supp(f)} f(x,y)\,dxdy = Arie(Supp(f))$$

$$m_{00} = \sum_{f(x,y)\neq 0}\sum 1 = Arie(Supp(f))$$

Momentele statistice nu prezinta nici un grad de invarianta. (depinde pozitia formei in imagine, de dimensiunea si orientarea acesteia).

Centrul de greutate

Coordonatele centrului de greutate al fomei, (μ_x, μ_y) se obtin prin:

$$\mu_x = \frac{m_{10}}{m_{00}}$$

$$\mu_y = \frac{m_{01}}{m_{00}}$$

Momente statistice centrate

asigura invarianta in raport cu translatia.

$$\mu_{pq} = \iint\limits_{Supp(f)} f(x,y)\left(x-\mu_x\right)^p\left(y-\mu_y\right)^q dxdy$$

$$\mu_{pq} = \sum_{f(x,y)\neq 0}\sum \left(x-\mu_x\right)^p\left(y-\mu_y\right)^q$$

Momente statistice centrate normalizate

asigura invarianta in raport cu translatia si scalarea.

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{(p+q+2)/2}}$$

Invariantii formei [Hu]

invarianti la translatie, scalare, rotatie, reflexie.

$$\Phi_1 = \eta_{20} + \eta_{02}$$

$$\Phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

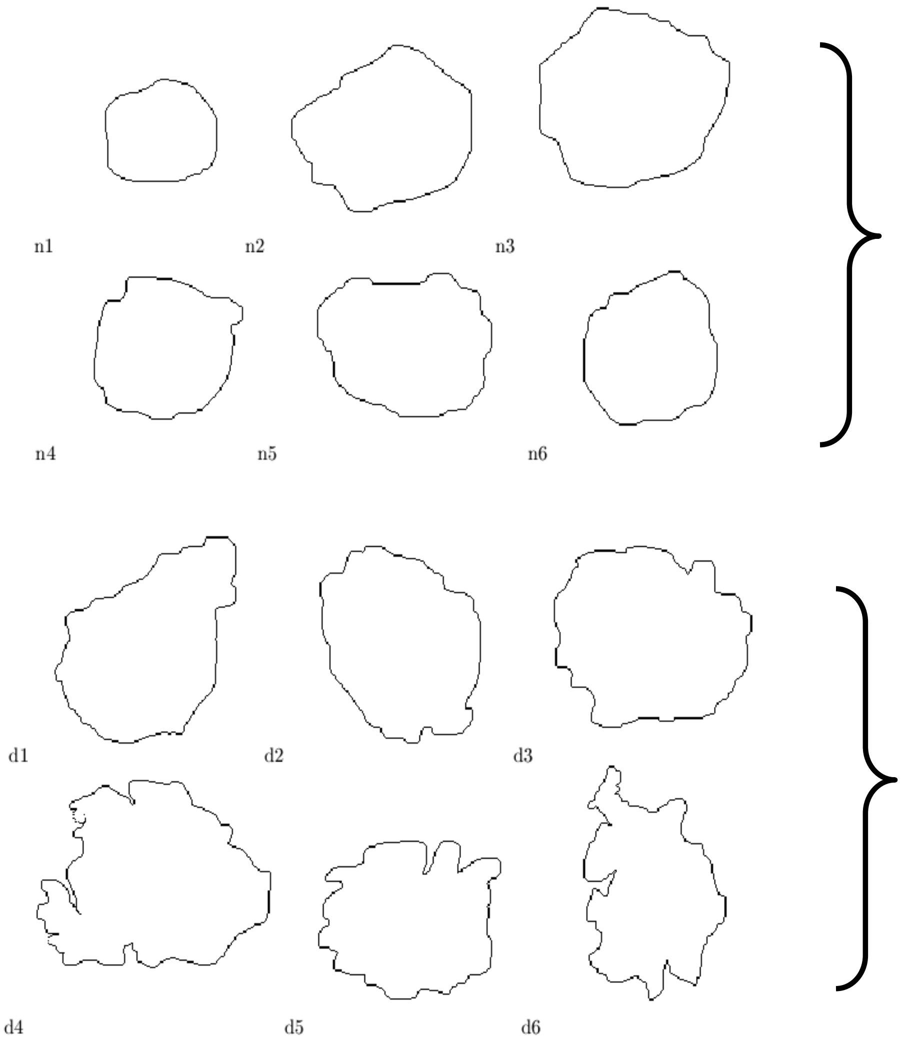
$$\Phi_3 = (\eta_{30} - 3\eta_{12})^2 + (\eta_{03} - 3\eta_{21})^2$$

$$\Phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{03} + \eta_{21})^2$$

$$\Phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{03} + \eta_{21})^2\right]$$
$$+ (\eta_{03} - 3\eta_{21})(\eta_{03} + \eta_{21})\left[(\eta_{03} + \eta_{21})^2 - 3(\eta_{30} + \eta_{12})^2\right]$$

$$\Phi_6 = (\eta_{20} + \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{03} + \eta_{21})^2\right] + 4\eta_{11}(\eta_{03} + \eta_{21})(\eta_{03} + \eta_{21})$$

Exemplu



n1,…,n6=normal

Formele difera prin dimensiune, orientare, iregularitate…

d1,…,d6=diabetic

Orientarea formei

Directia dupa care momentul de inertie al formei este minim.

$$\theta = \frac{1}{2}\arctan\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}$$

next:

Parametri de forma bazati pe contur

PARAMETRI DE FORMA:

DESCRIEREA CONTURURILOR

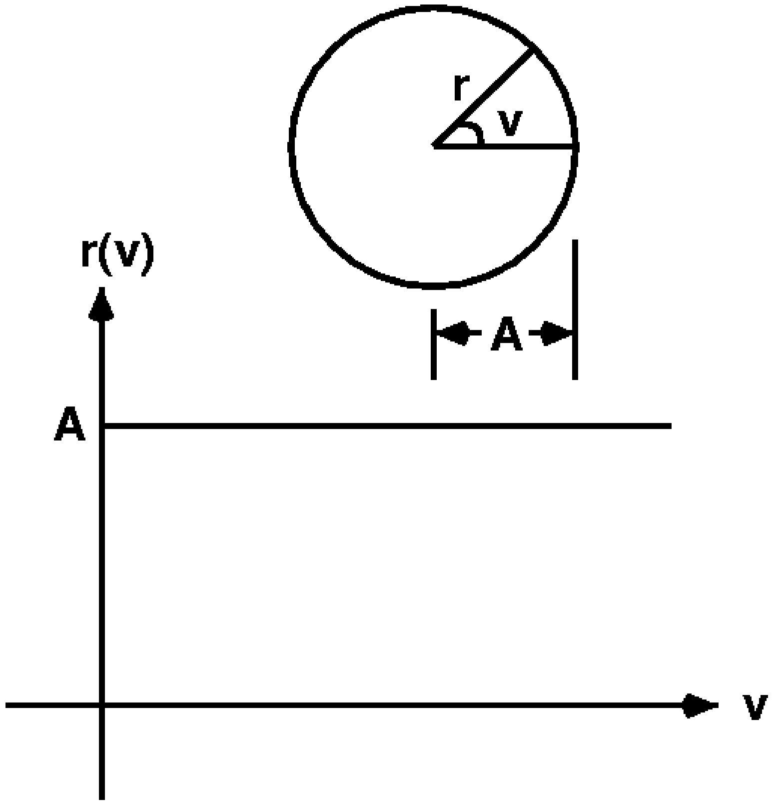
Parametri de forma

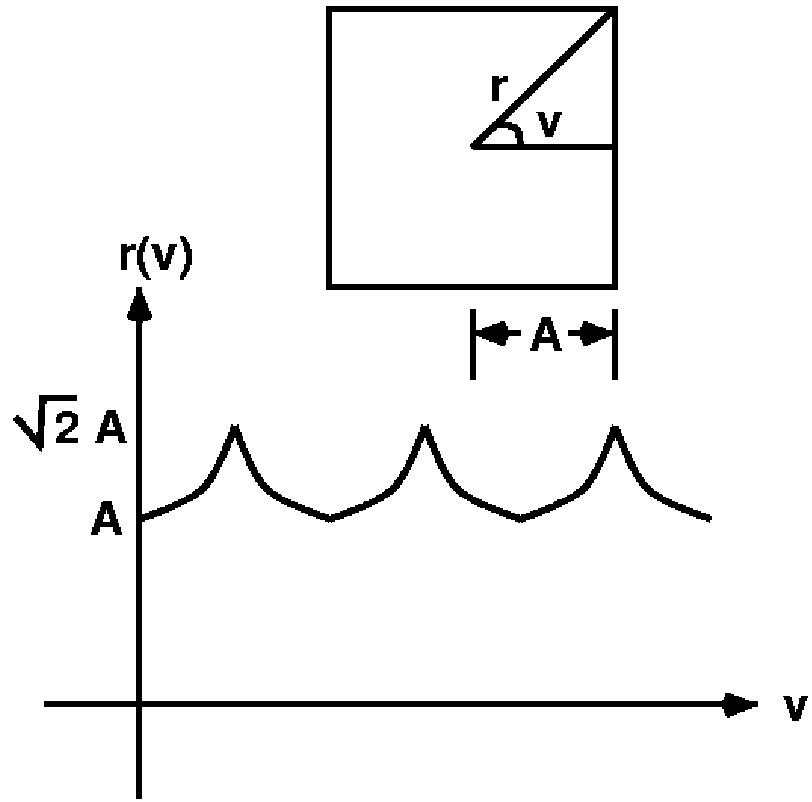
Asociaza unei forme (multime binara in planul 2D) un set de numere prin care aceasta poate fi recunoscuta, indiferent de pozitie, dimensiune, orientare.

Semnatura formei

- reprezentare functionla 1D a conturului
- abordare simpla: distanta de la un punct de referinta (de obicei centrul de greutate) ca functie de unghiul la centru
- frontiera 2D $\Rightarrow$ functie 1D
- probleme la rotatie si scalare
  - selectie centru
  - selectie punct de start
  - rescalare functie, de ex. valori $\in [0,1]$

Exemple de semnaturi





Descriptori Fourier de contur

- frontiera de K pixeli e reprezentata ca o secventa de coordonate
  - s(k)=(x(k),y(k)), k=0,1,2,...,K-1
  - numar complex s(k)=x(k)+iy(k)
  - 2D→1D
- DFT

$$a(u) = \frac{1}{K}\sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}, u = 0,1,2,\ldots,K-1$$

a(u) – descriptorii Fourier ai frontierei

Reconstructia formei din descriptorii Fourier

- reconstructie = DFT invers

$$s(k) = \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}, k = 0,1,2,..., K-1$$

- aproximarea frontierei daca se foloseste o serie truncheata de coeficienti (P<K)

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u) e^{j2\pi uk/K}, k = 0,1,2,..., K-1$$

  - frontiera va avea acelasi numar de pixeli
  - interpretare:
    - inalta frecventa = detalii fine
    - frecventa joase = forma in general

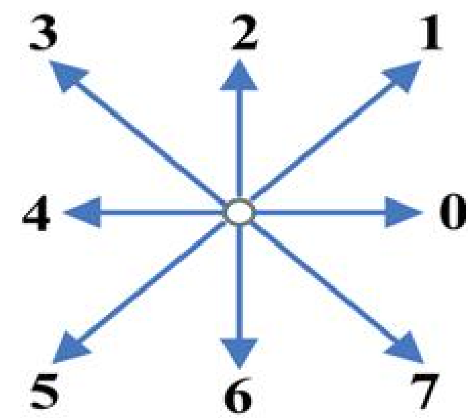
Proprietati

transformare	***frontiera***	***descriptori Fourier***
identitate	$s(k)$	$a(u)$
rotatie	$s_r(k)=s(k)e^{j\theta}$	$a_r(u)=a(u)e^{j\theta}$
translatie	$s_t(k)=s(k)+\Delta_{xy}$	$a_t(u)=a(u)+ \Delta_{xy}\delta(u)$
scalare	$s_s(k)=\alpha s(u)$	$a_s(k)=\alpha a(u)$
punct de start	$s_p(k)=s(k-k_0)$	$a_p(u)=a(u)e^{-j2\pi k_0 u/K}$

$\Delta_{xy}=\Delta x+j\Delta y$

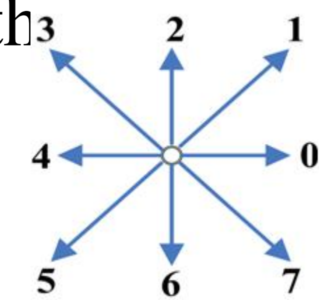
Chain Code

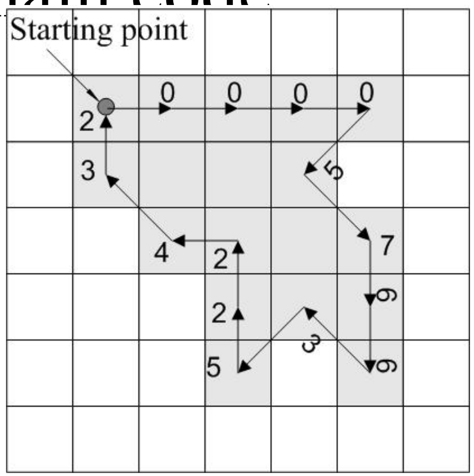
- What is chain code?
  - Representation of binary images (i.e. text images)
  - Traversing the edges in steps and encoding each step
- Freeman Chain Code (FCC)
  - Contains 8 codes
  - Each represents a direction between two pixels
  - Coding depends on the direction



Freeman Chain Code

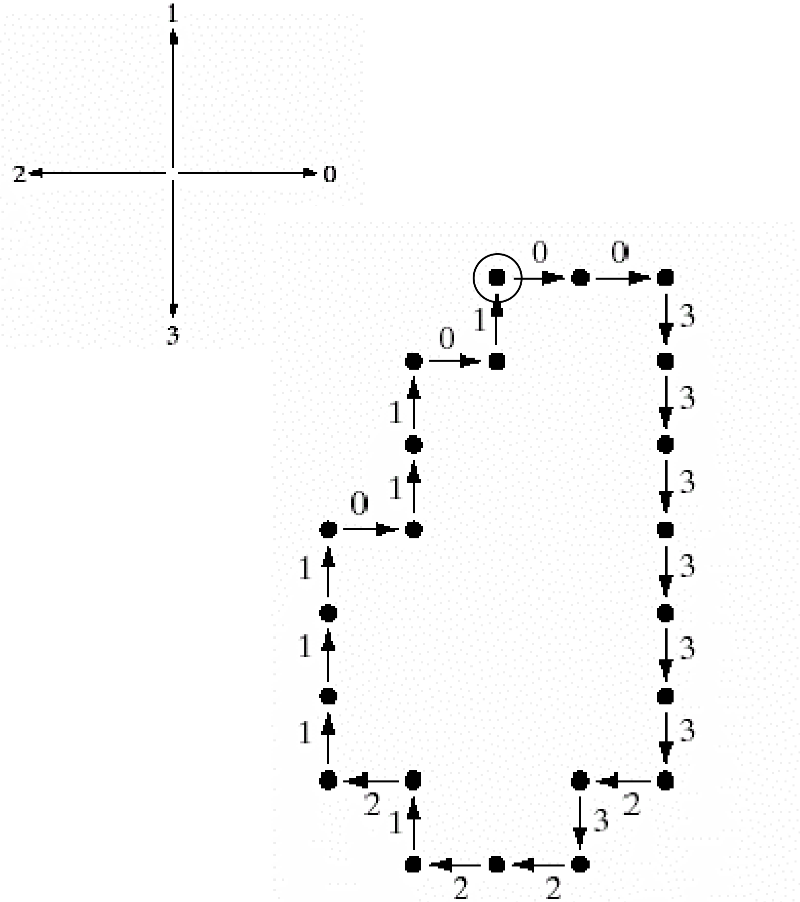
- **Freeman Chain Code Example**
  - Locate the first pixel (Start from the farth
    North West) and store its coordinates
  - Find the link to the next pixel
  - Encode based on Freeman chain code
  - Continue that way …



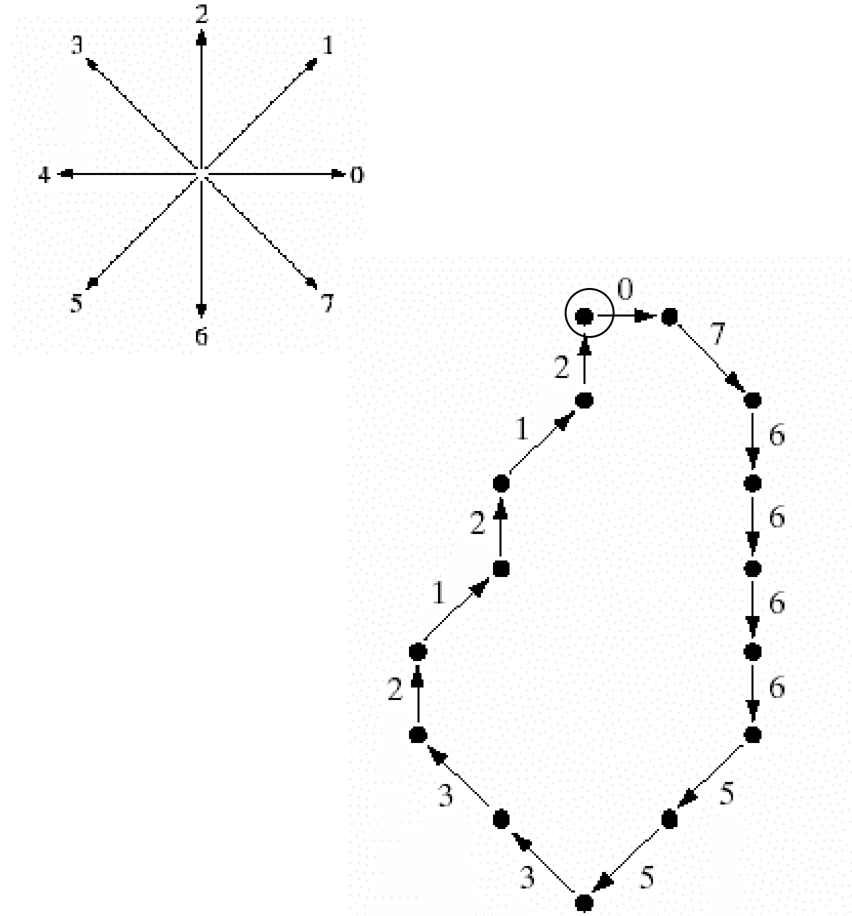


- Result:
  - 0, 0, 0, 0, 5, 7, 6, 6, 3, 5, 2, 2, 4, 3, 2

Chain Codes



4-directional chain code:
0033333323221211101101



8-directional chain code:
076666553321212

Hough Transform

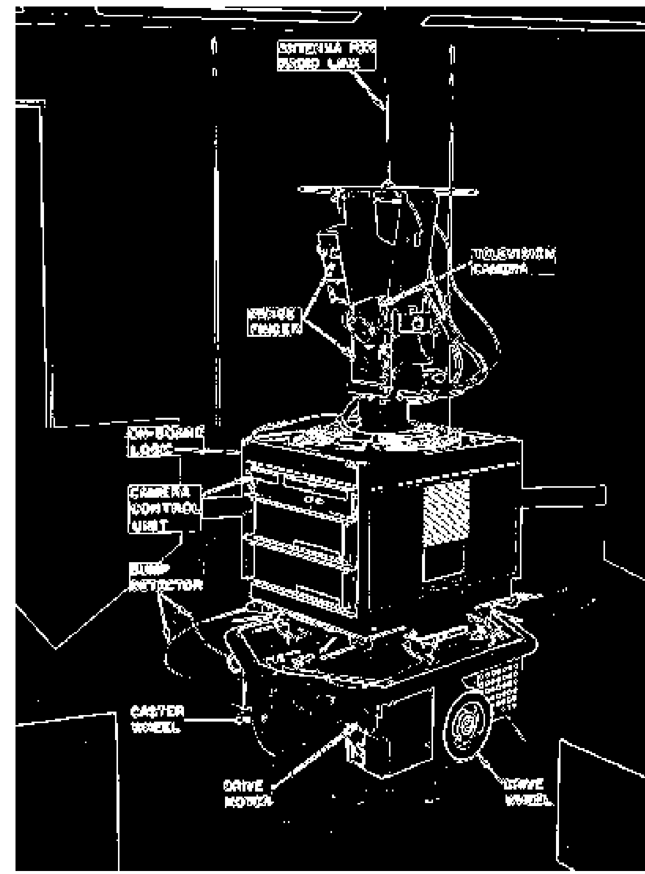
Jeremy Wyatt

Finding edge features

But we haven't found edge segments, only edge points

How can we find and describe more complex features?

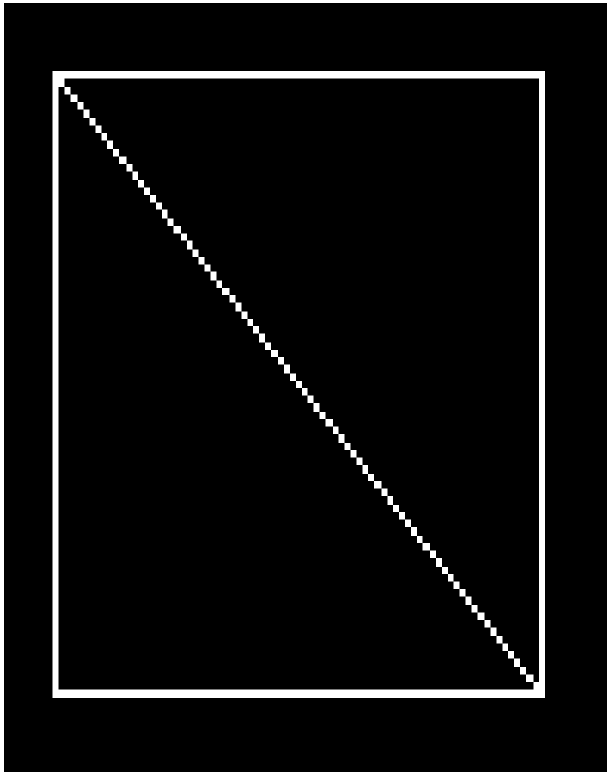
The Hough transform is a common approach to finding parameterised line segments (here straight lines)



The basic idea

Each straight line in this image can be described by an equation

Each white point if considered in isolation could lie on an infinite number of straight lines



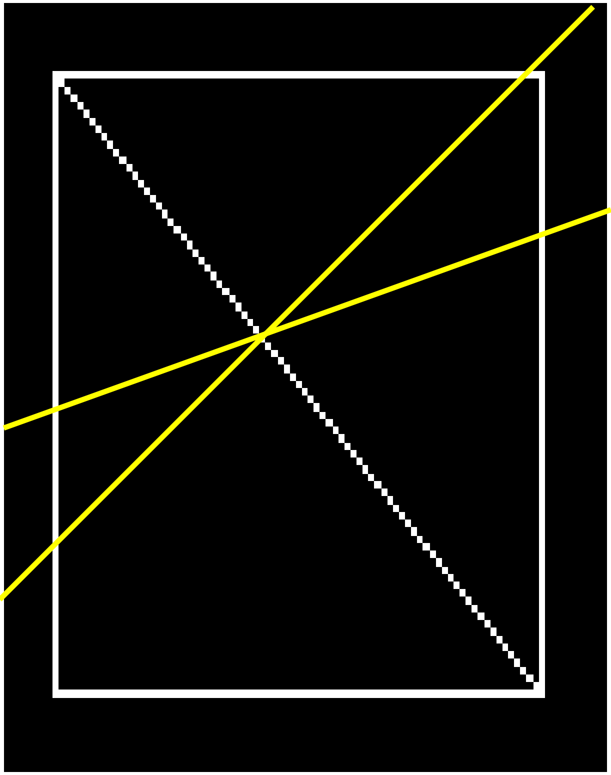
The basic idea

Each straight line in this image can be described by an equation

Each white point if considered in isolation could lie on an infinite number of straight lines

In the Hough transform each point votes for every line it could be on

The lines with the most votes win



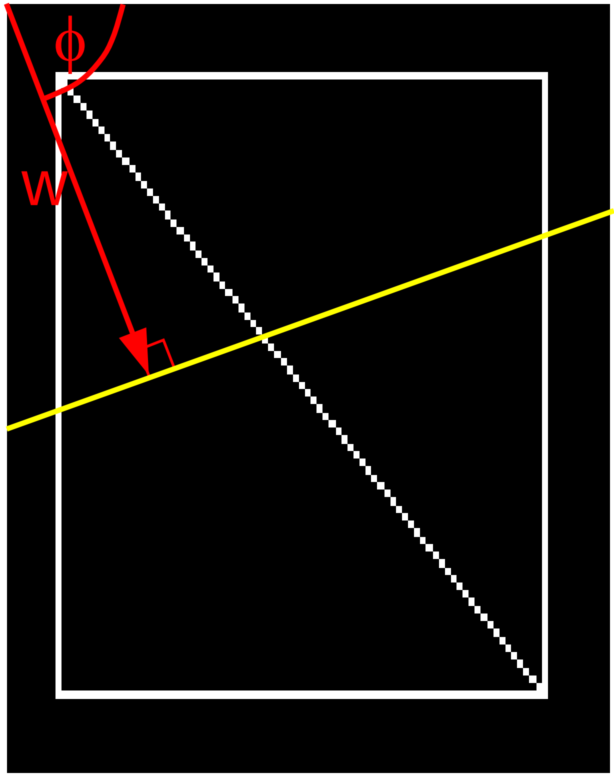
How do we represent lines?

Any line can be represented by two numbers

Here we will represent the yellow line by (w,ϕ)

In other words we define it using

- a line from an agreed origin
- of length w
- at angle ϕ to the horizontal

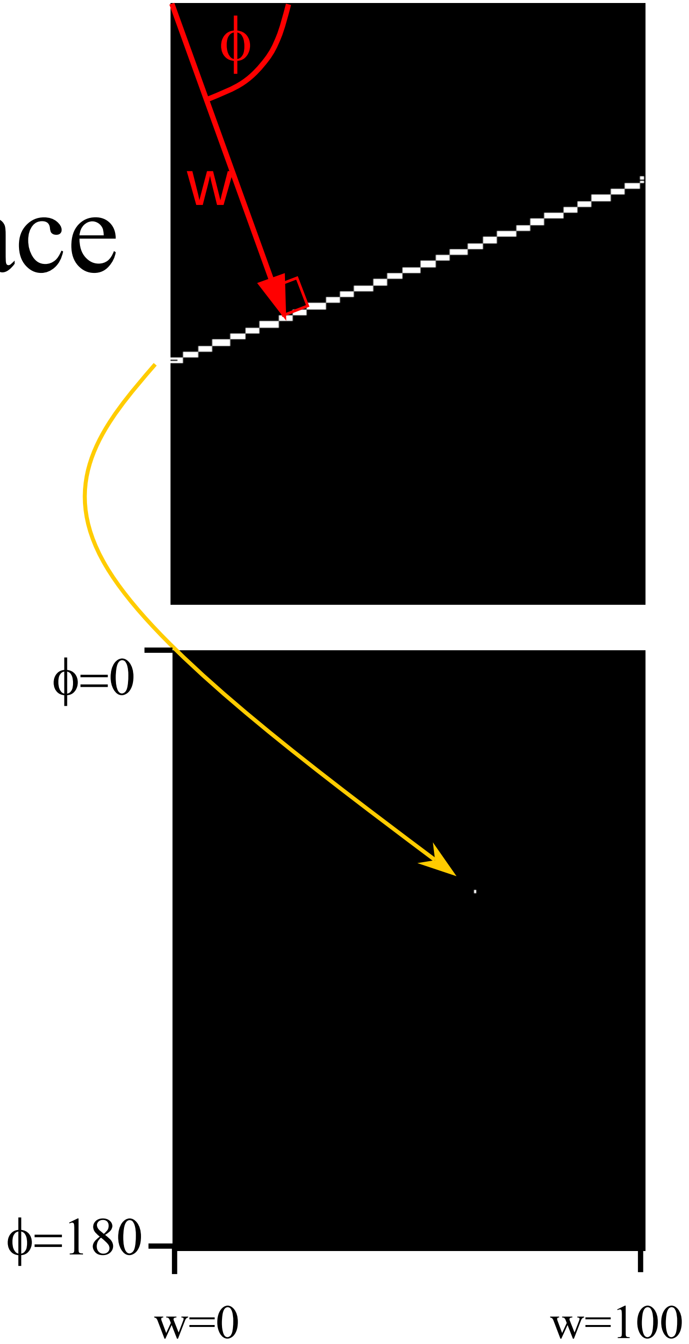


Hough space

Since we can use (w,ϕ) to represent any line in the image space

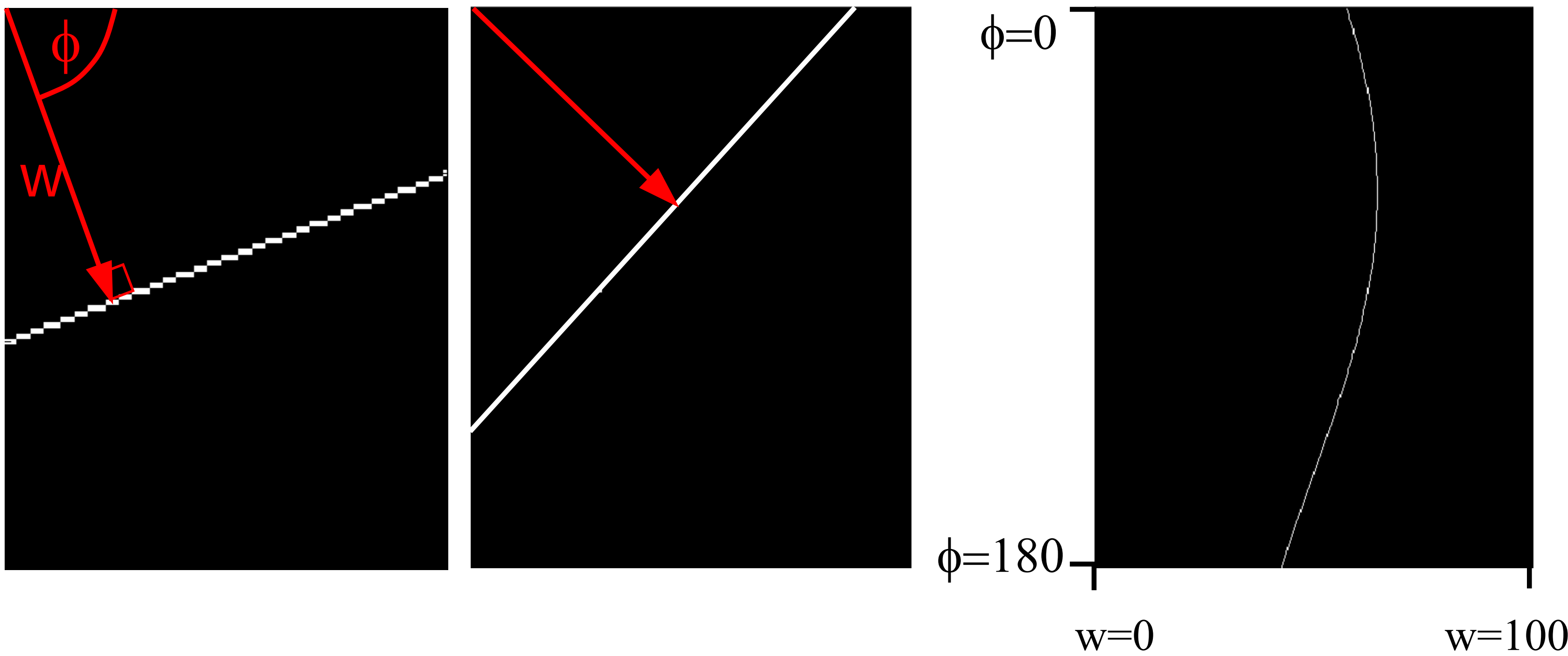
We can represent any line in the image space as a point in the plane defined by (w,ϕ)

This is called Hough space



How does a point in image space vote?

$$w = x\cos(\phi) + y\sin(\phi)$$



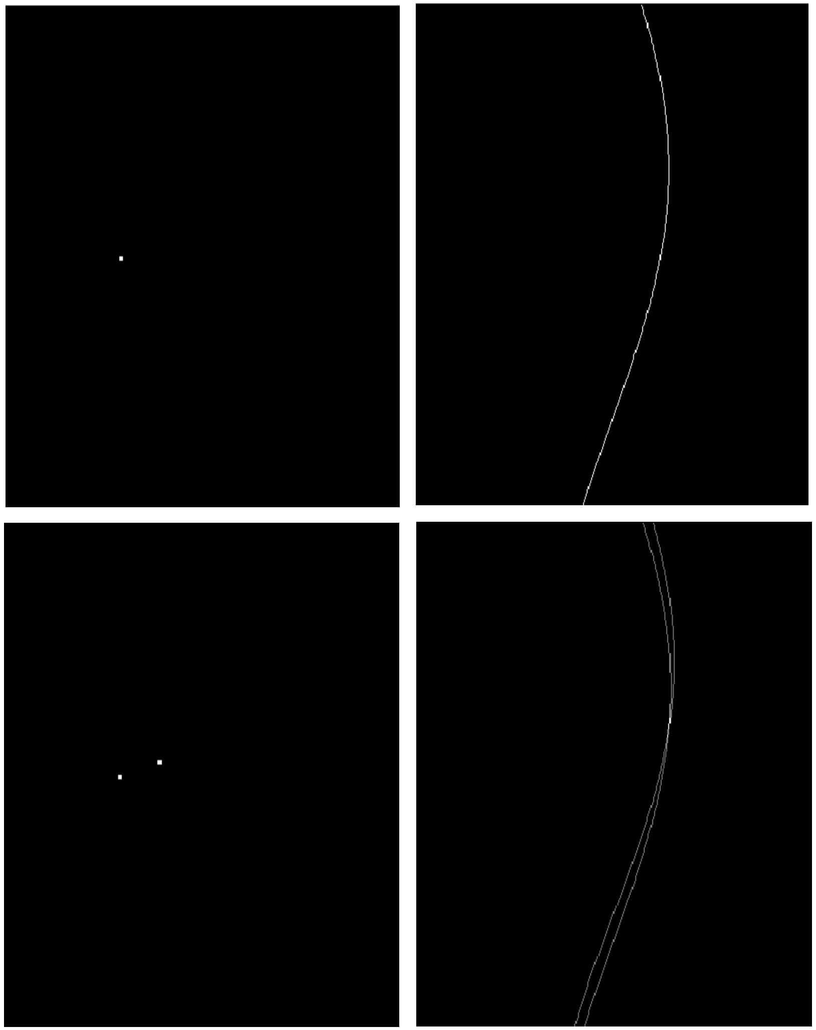
How do multiple points prefer one line?

One point in image space corresponds to a sinusoidal curve in image space

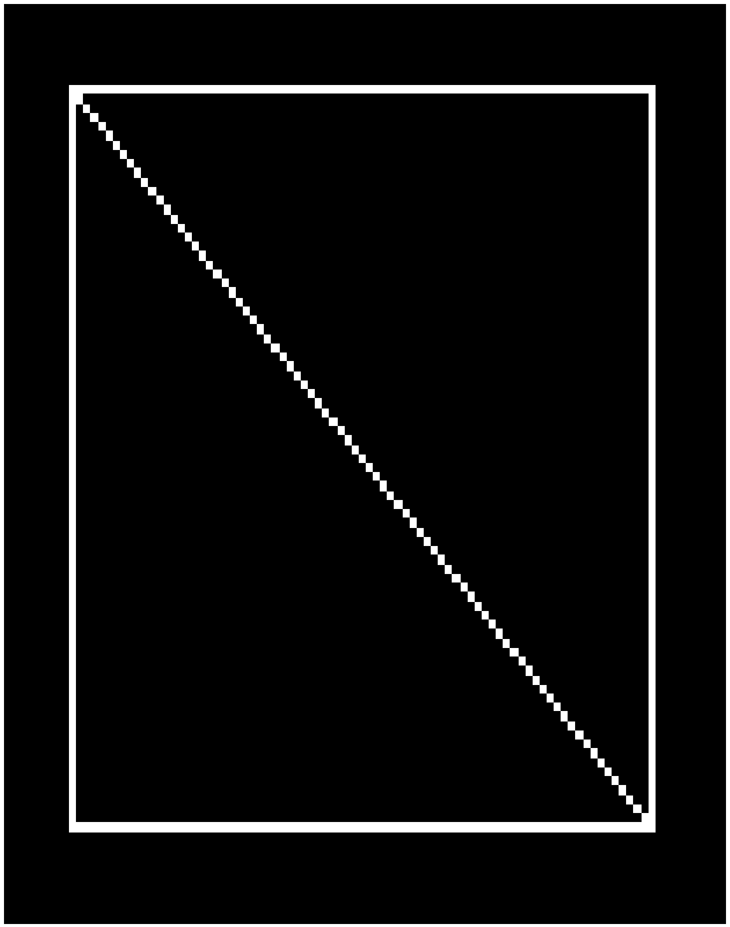
Two points correspond to two curves in Hough space

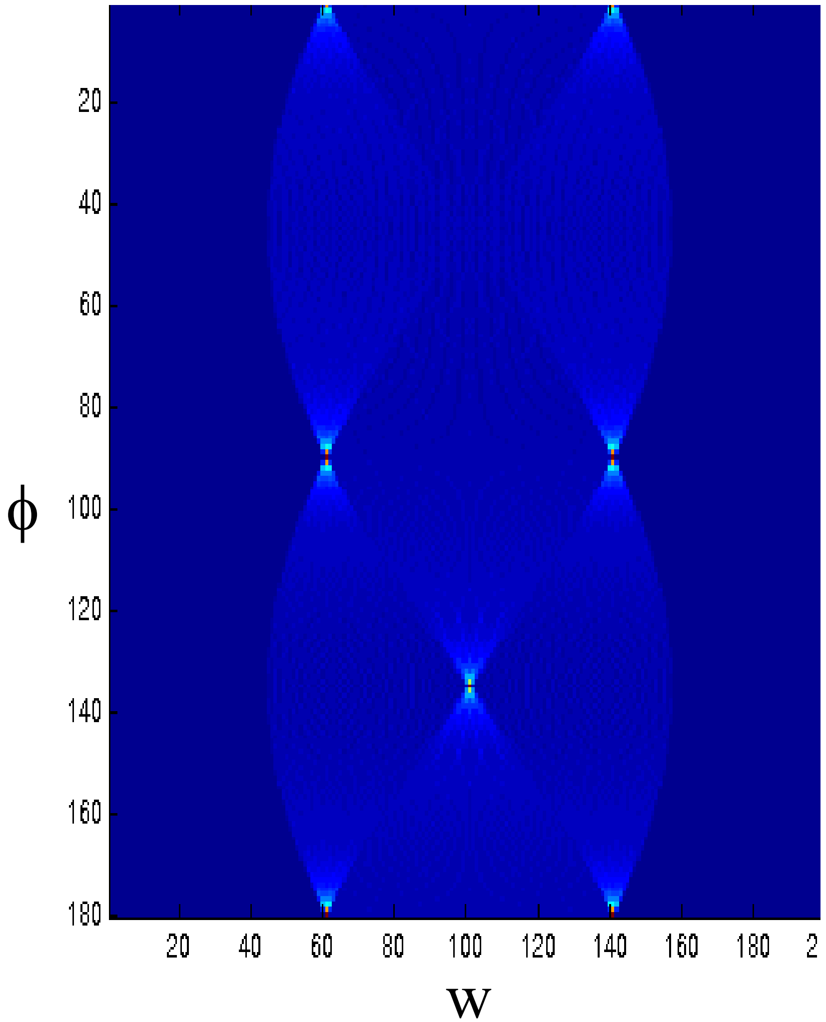
The intersection of those two curves has “two votes”.

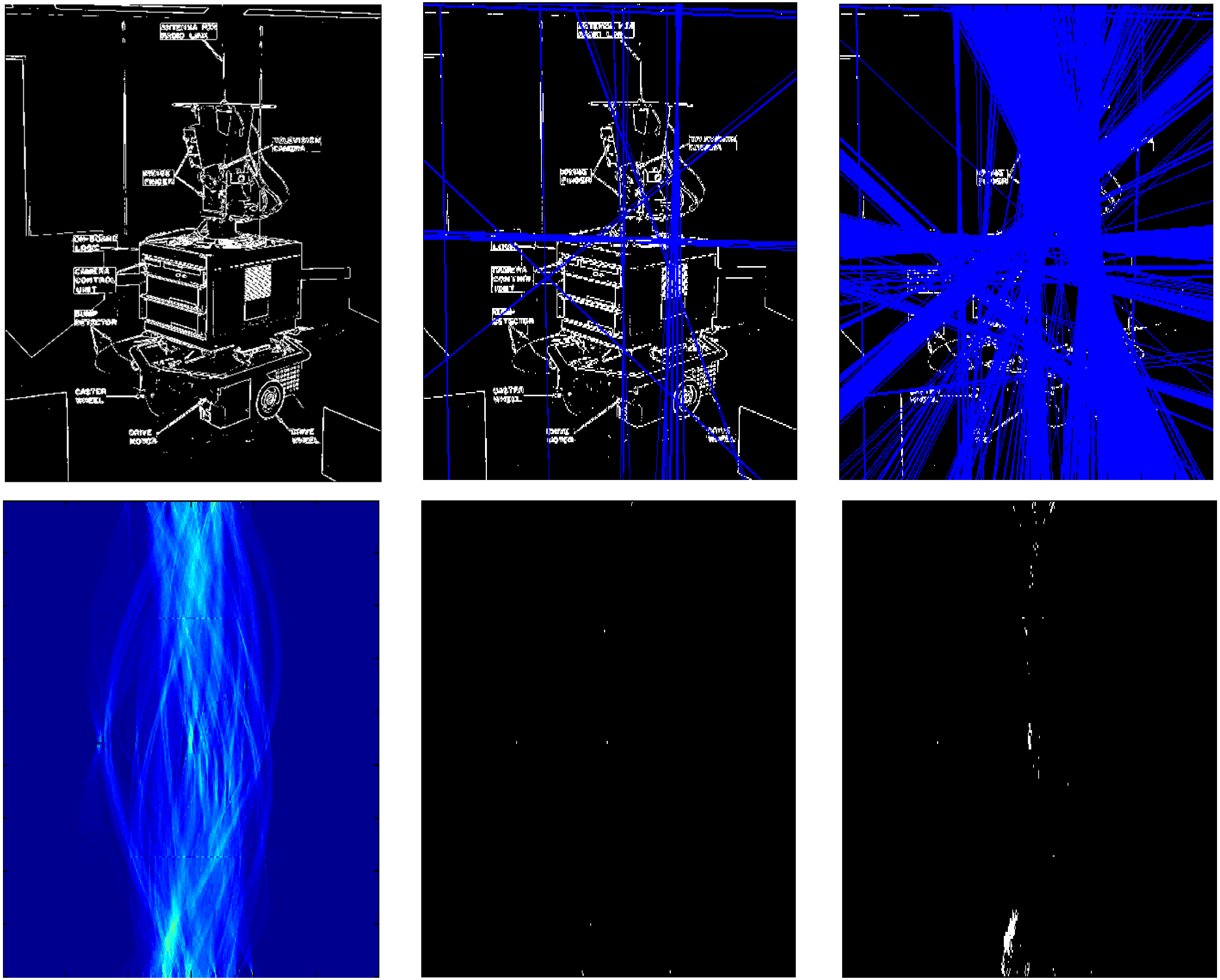
This intersection represents the straight line in image space that passes through both points



A simple example







Hough Transform

- There are generalised versions for ellipses, circles
- For the straight line transform we need to supress non-local maxima
- The input image could also benefit from edge thinning
- Single line segments not isolated
- Will still fail in the face of certain textures