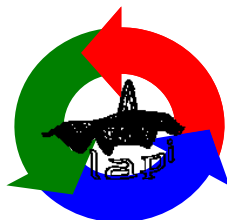


SEGMENTAREA IMAGINILOR (ORIENTATA PE REGIUNI)

THRESHOLDING

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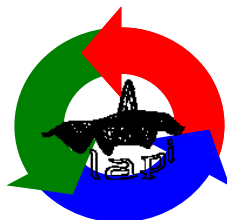
Cea mai simpla caracteristica: **nivelul de gri**

Presupunem ca nivelul de gri este reprezentativ si suficient pentru caracterizarea tipurilor de obiecte din imagine.

Trebuie deci identificate “concentrarile” de nivele de gri, adica **modurile din histograma imaginii**. Fiecare mod bine identificat va corespunde unui tip de obiecte din imagine.

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Histograma

Histograma = functie ce asociaza fiecarui nivel de gri posibil probabilitatea [sa] de aparitie in imagine.

$h(u)$ = numar pixeli de nivel de gri “ u ” / numar total pixeli

$$h(u) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \delta(f(m,n) - u), \quad u = 0, 1, \dots, L-1$$

Histograma este o functie de densitate de probabilitate.

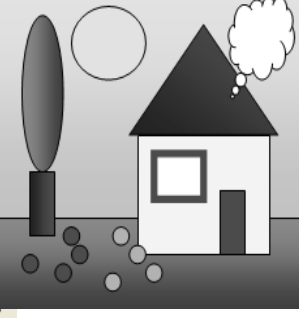
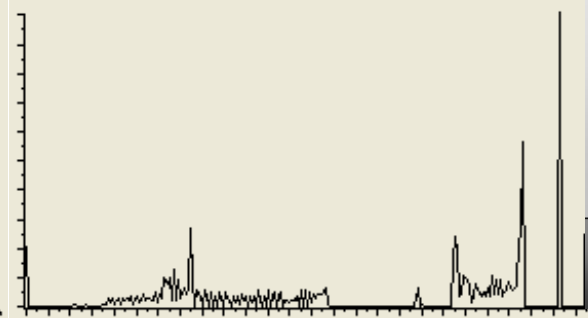
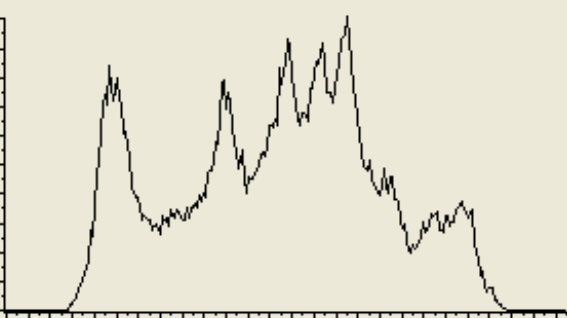
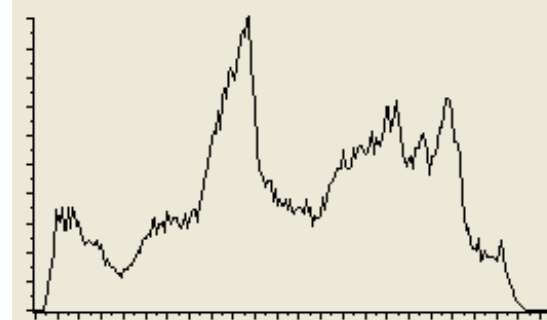
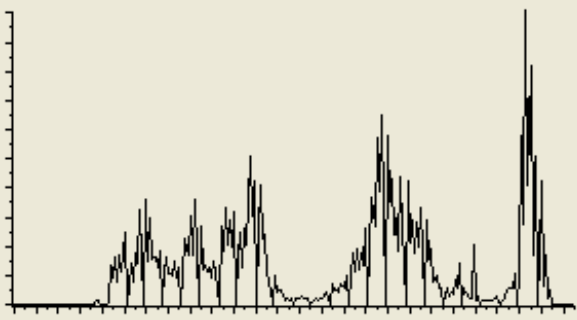
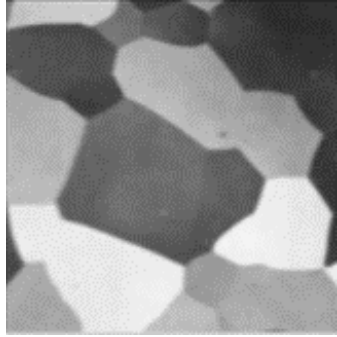
$$\sum_{u=0}^{L-1} h(u) = 1$$

Histograma descrie continutul “de culoare/ de gri” al imaginii.

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Histograma



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Segmentarea pe histograma = Thresholding (praguire)

gasirea “pragurilor” de separare dintre modurile histogramei de nivele de gri a imaginii.

Fie T_k pragurile de segmentare pe histograma.

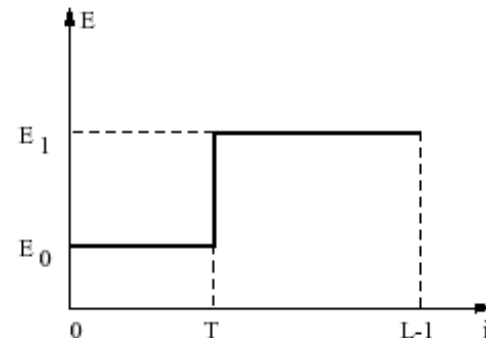
$$g(m,n) = E_k, \text{ daca } T_k \leq f(m,n) \leq T_{k+1}$$

E_k este eticheta ce se atribuie tipului de obiecte k

$$T_0 = 0, T_C = L, k = 0, 1, \dots, L-1$$

Caz particular : $C = 2$ (binarizarea)

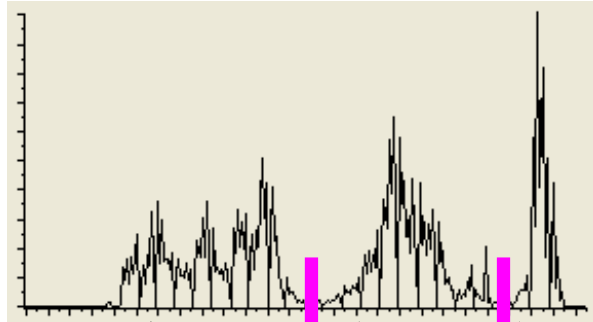
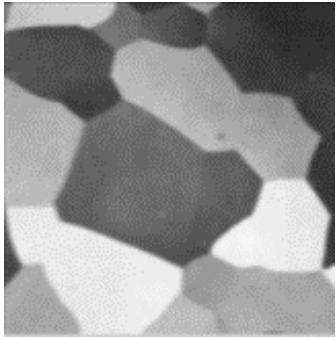
$$g(m,n) = \begin{cases} E_0, & f(m,n) \leq T \\ E_1, & f(m,n) > T \end{cases}$$



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Evident, alegerea pragurilor de segmentare T_k este cruciala.



obiecte
intunecate

obiecte
gri mediu

obiecte foarte luminoase

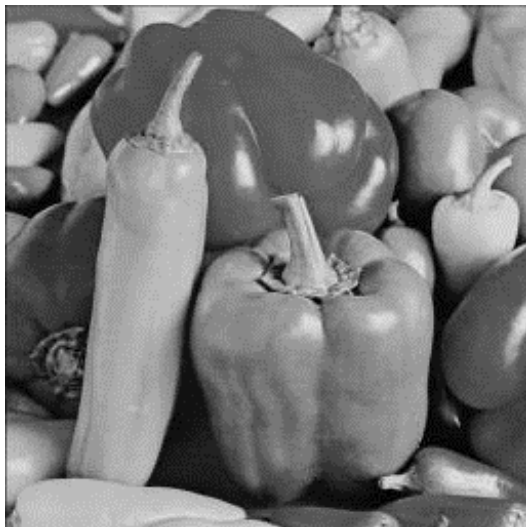
Pragurile se aleg pe minimele histogramei (separatia dintre moduri).

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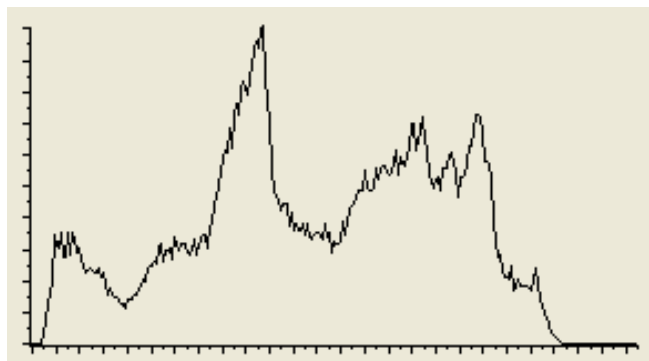
Exemplu



$$C=3$$

$$T_1=40$$

$$T_2=100$$



$$C=4$$

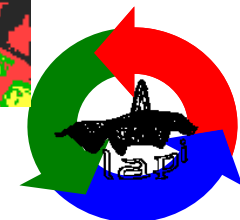
$$T_1=40$$

$$T_2=100$$

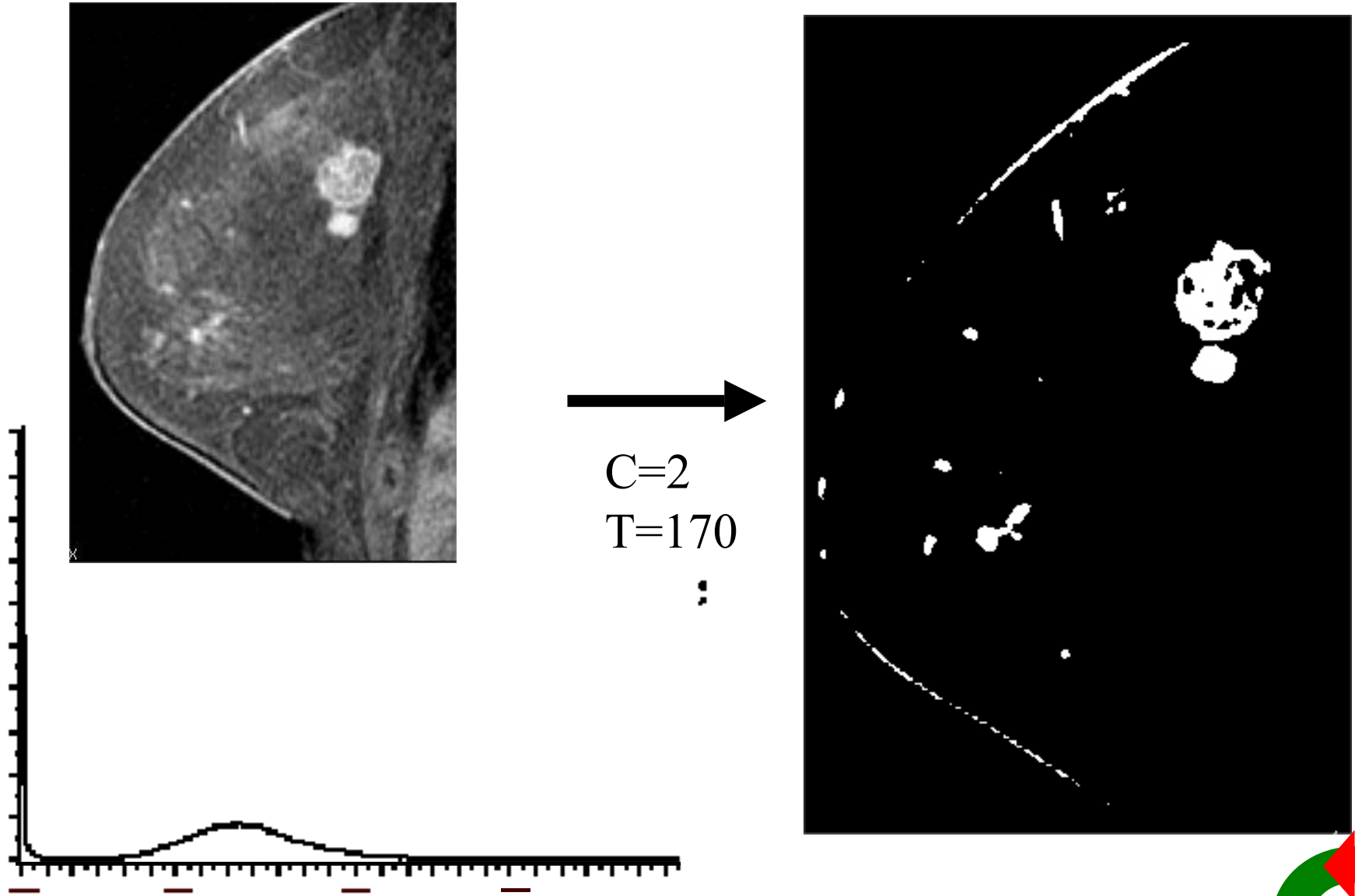
$$T_3=$$



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Exemplu



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Segmentare

Evident, segmentarea inseamna ca pentru fiecare pixel din imagine, in functie de valoarea acestuia (si poate si alte valori din imagine) si in functie de parametri specifici unui anume metode, sa se decida care componenta din care respectivul pixel face parte.

Parametrii care particularizeaza o metoda generala pentru o anume imagine pot sa depinda

doar de valoare pixelilor

segm. globala

de valoarea pixelilor si de caracteristici
de vecinatate ale pixelilor

segm. locala

de valoarea pixelilor, de caracteristici
de vecinatate ale pixelilor si de pozitia
in imagine

segm. dinamica

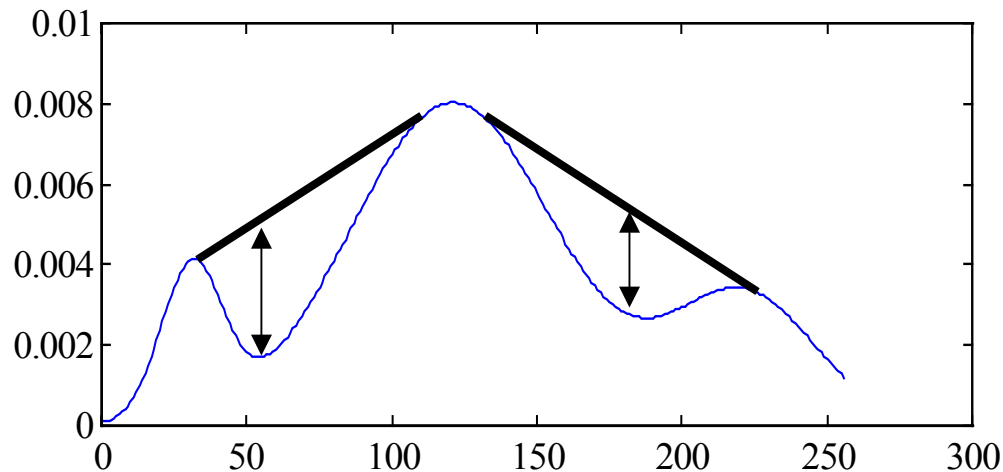
segm. adaptiva

Segmentare globala

Metoda concavitatilor histogramei

Minimele histogramei sunt vazute ca niste concavitati ale functiei $h(i)$.

Determinam concavitatile ca diferenta fata de anvelopa convexa a functiei histograma.



masura de “echilibru”
a pragului de segmentare
fata de diferentele de
concavitate

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Segmentare globala

Metode bazate pe histograma cumulativa

$$H(j) = \sum_{i=1}^j h(i), \quad j = 0, 1, \dots, L - 1.$$

functia de rapartitie asociata
histogramei

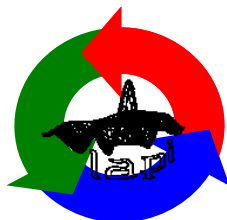
Presupunand ca obiectele de interes sunt de nivel de gri inchis si ocupa o arie relativa $P\%$ din imagine, atunci pragul de segmentare T se determina prin:

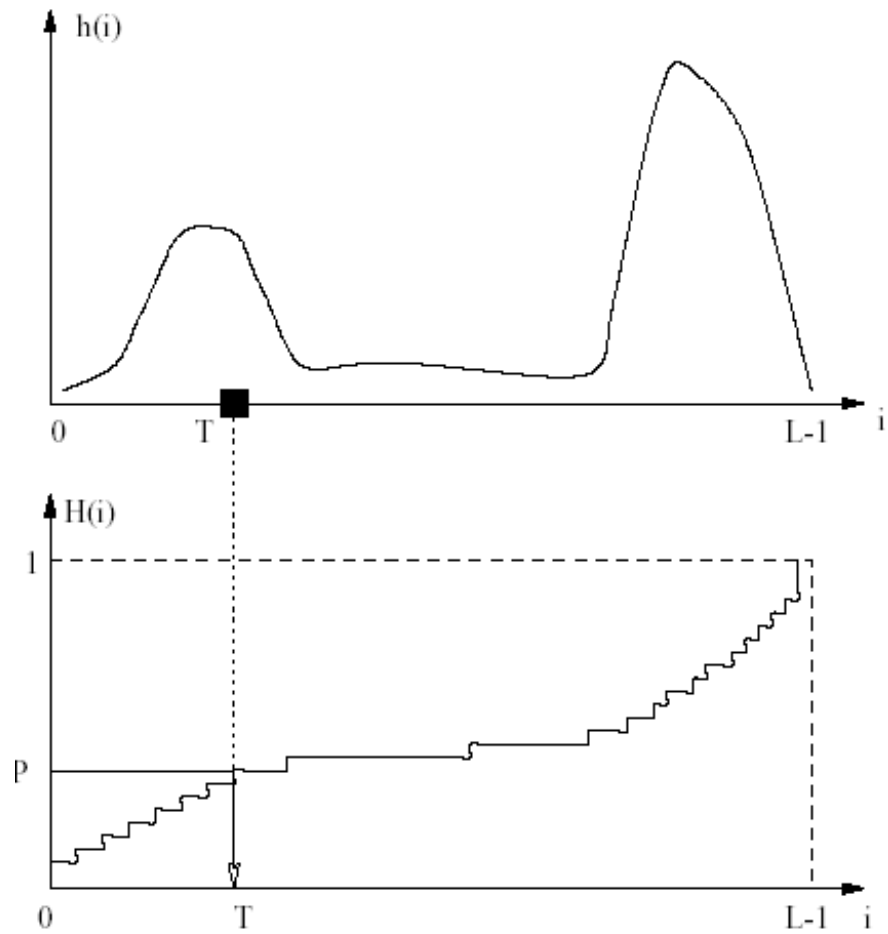
$$H(T) \cong P.$$

Pentru obiecte de interes luminoase,

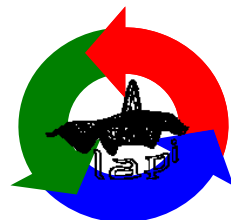
$$H(T) \cong 1 - P$$

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Segmentare globala

Metode bazate pe histograma cumulativa

Pragurile se aleg in punctele de curbura 0 ale functiei de repartitie a imaginii.

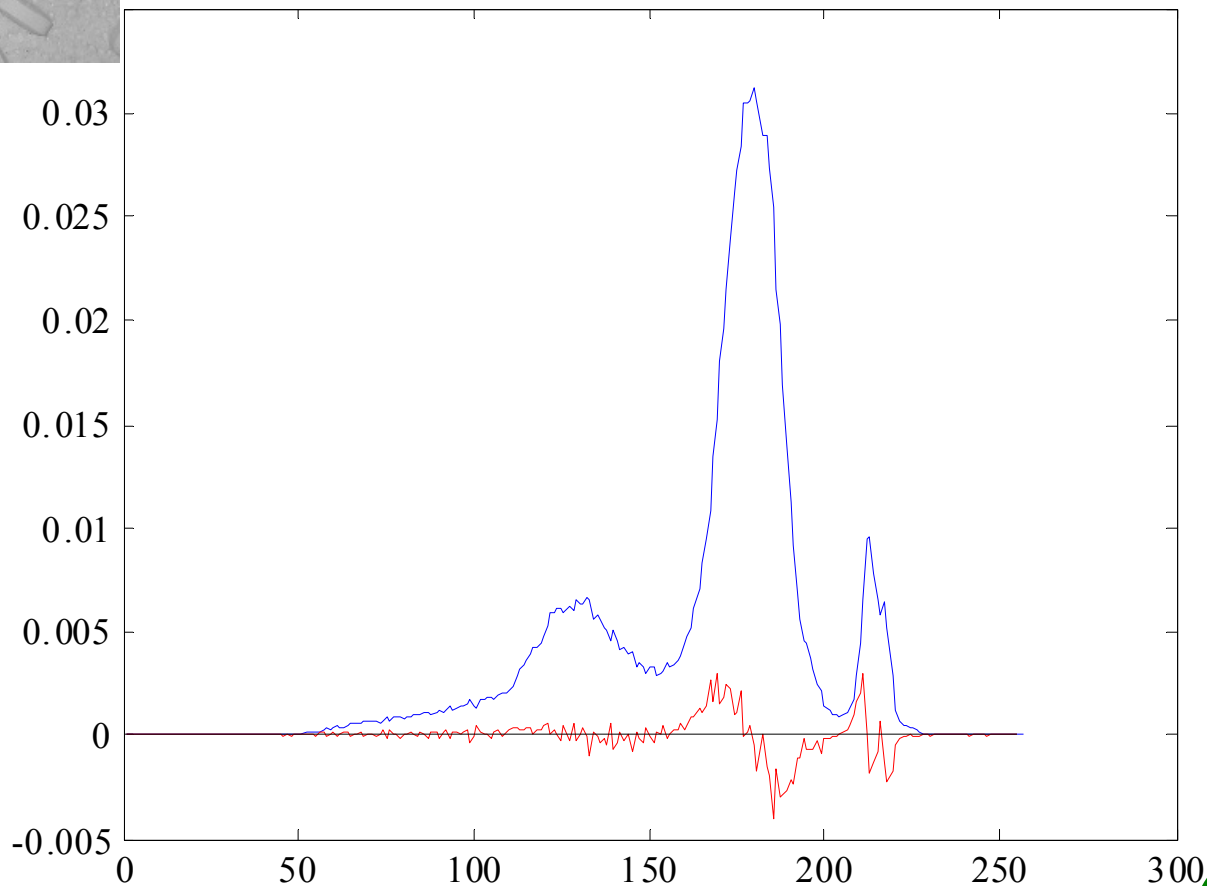
$$C(x) = H''(x) \left(1 + H'^2(x)\right)^{-3/2}$$

$$C(x) = h'(x) \left(1 + h^2(x)\right)^{-3/2}$$

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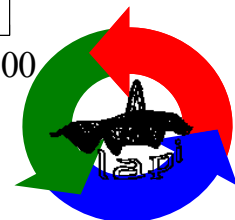
LABORATORUL DE ANALIZA ȘI PRELUCRAREA IMAGINILOR





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LABORATORUL DE ANALIZA ȘI PRELUCRAREA IMAGINILOR

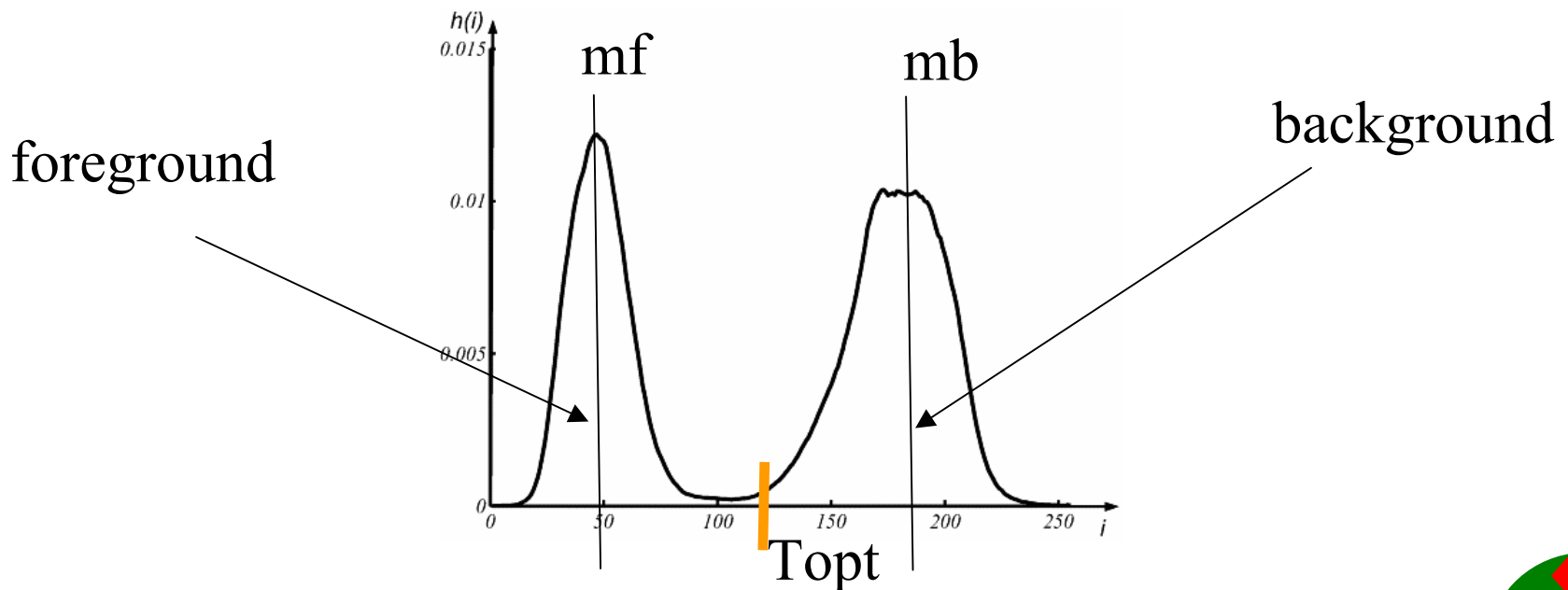


Riddler

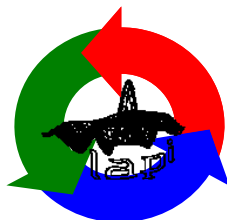
$$T_{\text{opt}} = \lim_{n \rightarrow \infty} \frac{m_f(T_n) + m_b(T_n)}{2}$$

cu

$$m_f(T_n) = \sum_{g=0}^{T_n} gp(g) \quad m_b(T_n) = \sum_{g=T_n+1}^G gp(g)$$



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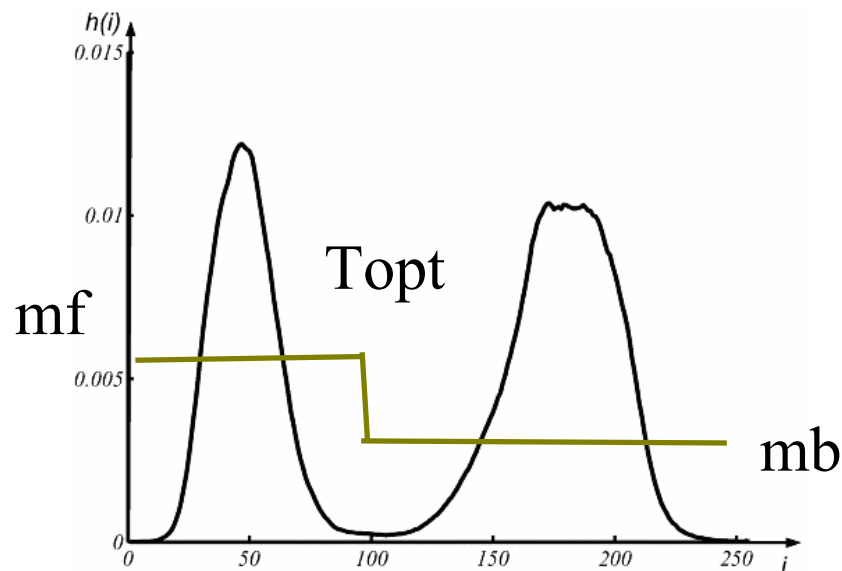


Ramesh et al – gasirea pragului prin aproximarea formei

$$T_{\text{opt}} = \min \left[\sum_{g=0}^T [b_1(T) - g]^2 + \sum_{g=T+1}^G [b_2(T) - g]^2 \right]$$

where $b_1(T) = m_f(T)/P(T)$, $b_2(T) = m_b(T)[1 - P(T)]$

presupune aproximarea histogramei cu functie treapta;
inaltimea treptelor sunt mediile valorilor histogramei
de o parte si de alta a pragului.



Segmentare globala

Determinarea pragurilor ca problema de cuantizare

Cuantizare = aproximarea unui set de valori dat (in general un interval de valori) cu un numar fixat de valori din aceeași gama, in sensul minimizării unei erori de aproximare.

Cuantizarea optimala Lloyd-Max:

Cuantizarea trebuie sa minimize eroarea patratica medie de aproximare a dateor cu valorile cuantizate.

Necunoscute: valori de aproximare, intervale de cuantizare

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Lloyd-Max

$$\varepsilon = \sum_{i=1}^L \int_{x_i}^{x_{i+1}} (x - y_i)^2 w_\xi(x) dx.$$

$$1. \quad \frac{\partial \varepsilon}{\partial x_k} = 0 \quad \blacktriangleright \quad 2(x_k - y_{k-1})^2 w_\xi(x_k) - 2(x_k - y_k)^2 w_\xi(x_k) = 0.$$

\blacktriangledown

$$x_k = \frac{y_{k-1} + y_k}{2}$$

$$2. \quad \frac{\partial \varepsilon}{\partial y_k} = 0 \quad \blacktriangleright \quad -2 \int_{x_k}^{x_{k+1}} (x - y_k) w_\xi(x) dx = 0. \quad \blacktriangleright \quad y_k = \frac{\int_{x_k}^{x_{k+1}} x w_\xi(x) dx}{\int_{x_k}^{x_{k+1}} w_\xi(x) dx}$$

$$y_k = \int_{-\infty}^{\infty} x w_{\xi|\{x_k \leq \xi \leq x_{k+1}\}}(x) dx = \bar{\xi}|\{x_k \leq \xi \leq x_{k+1}\}$$

Pentru segmentare, numarul de valori de aproximare este numarul de clase de obiecte dorite, valorile de aproximare (ce vor constitui etichetele) sunt mijloacele intervalelor claselor (si sunt putin importante) iar pragurile de segmentare corespund limitelor intervalelor de cuantizare.

$$\begin{aligned}
 \varepsilon &= \sum_{i=1}^L \int_{x_i}^{x_{i+1}} (x - \bar{\xi}_{\{x_i \leq \xi \leq x_{i+1}\}})^2 w_{\xi}(x) dx = \\
 &= \sum_{i=1}^L \underbrace{\int_{x_i}^{x_{i+1}} w_{\xi}(z) dz}_{P(x_i \leq \xi \leq x_{i+1})} \underbrace{\int_{x_i}^{x_{i+1}} (x - \bar{\xi}_{\{x_i \leq \xi \leq x_{i+1}\}})^2 \frac{w_{\xi}(x)}{\int_{x_i}^{x_{i+1}} w_{\xi}(z) dz} dx}_{\sigma_{\xi|\{x_i \leq \xi \leq x_{i+1}\}}^2} \\
 &= \sum_{i=1}^L P(x_i \leq \xi \leq x_{i+1}) \sigma_{\xi|\{x_i \leq \xi \leq x_{i+1}\}}^2
 \end{aligned}$$

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Aceasta revine la a minimiza varianta medie a partitiei, adica suma variantelor claselor partitiei ponderate cu probabilitatile lor de aparitie.

Caracterizarea partitionarii bune prin generarea de moduri “inguste” se regaseste in majoritatea metodelor de alegere a pragurilor de segmentare, chiar daca formulata diferit.

Exemple clasice sunt metodele Otsu si respectiv Fisher.

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LABORATORUL DE ANALIZA ȘI PRELUCRAREA IMAGINILOR



Segmentare globala

Metoda Fisher

Partitionarea optima minimizeaza momentele de inertie ale claselor fata de media lor.

$$I_k^2 = \sum_{i=T_{k-1}}^{T_k} h(i) (i - \mu_k)^2.$$

$$J = \sum_{k=1}^C I_k^2 = \sum_{k=1}^C \sum_{i=T_{k-1}}^{T_k} h(i) (i - \mu_k)^2.$$

$$J = \sum_{a=0}^{L-1} a^2 h(a) - \sum_{i=1}^C \frac{\left(\sum_{a=\alpha_i}^{\alpha_{i+1}-1} ah(a) \right)^2}{\left(\sum_{a=\alpha_i}^{\alpha_{i+1}-1} h(a) \right)^2}$$

de minimizat

$$\bar{J} = \sum_{i=1}^C \frac{\left(\sum_{a=\alpha_i}^{\alpha_{i+1}-1} ah(a) \right)^2}{\left(\sum_{a=\alpha_i}^{\alpha_{i+1}-1} h(a) \right)^2} = \sum_{k=1}^C P_k \mu_k^2.$$

de maximizat

Segmentare globala

Metoda Otsu

Partitionarea optima minimizeaza unul dintre rapoartele dintre

- varianta partitiei si varianta globala

-

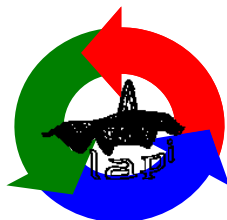
Cel mai simplu: minimizarea
variantei medii globale de partitionare

$$J = \sum_{k=1}^C P_k \sigma_k^2$$

In acest caz, metoda Otsu e identica cu cuantizarea optimala.

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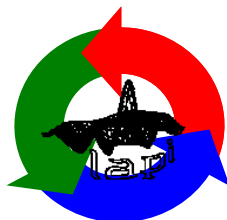


Implementare:

Cel mai simplu in cazul binarizarii (un prag) - functia criteriu de optimizat este calculata pentru toate valorile posibile ale nivelelor de gri, cu valorile statistice ce corespund claselor (probabilitati, medii, variante) evaluate iterativ.

$$P_1(T_1 + 1) = P_1(T_1) + h(T_1 + 1)$$
$$\mu_1(T_1 + 1) = \frac{P_1(T_1)\mu_1(T_1) + (T_1 + 1)h(T_1 + 1)}{P_1(T_1 + 1)}$$

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Segmentarea cu prag optim

Sa presupunem cunoscute: numarul de tipuri de obiecte din imagine, proportiile in care acestea ocupa suprafata imaginii si distributia nivelelor de gri caracteristice fiecarui tip de obiect.

$$h(x) = \sum_{i=1}^C P_i p_i(x)$$

$$\sum_{i=1}^C P_i = 1$$

Pentru binarizare $C=2$:

$$h(x) = P_1 p_1(x) + P_2 p_2(x)$$

$$P_1 + P_2 = 1$$

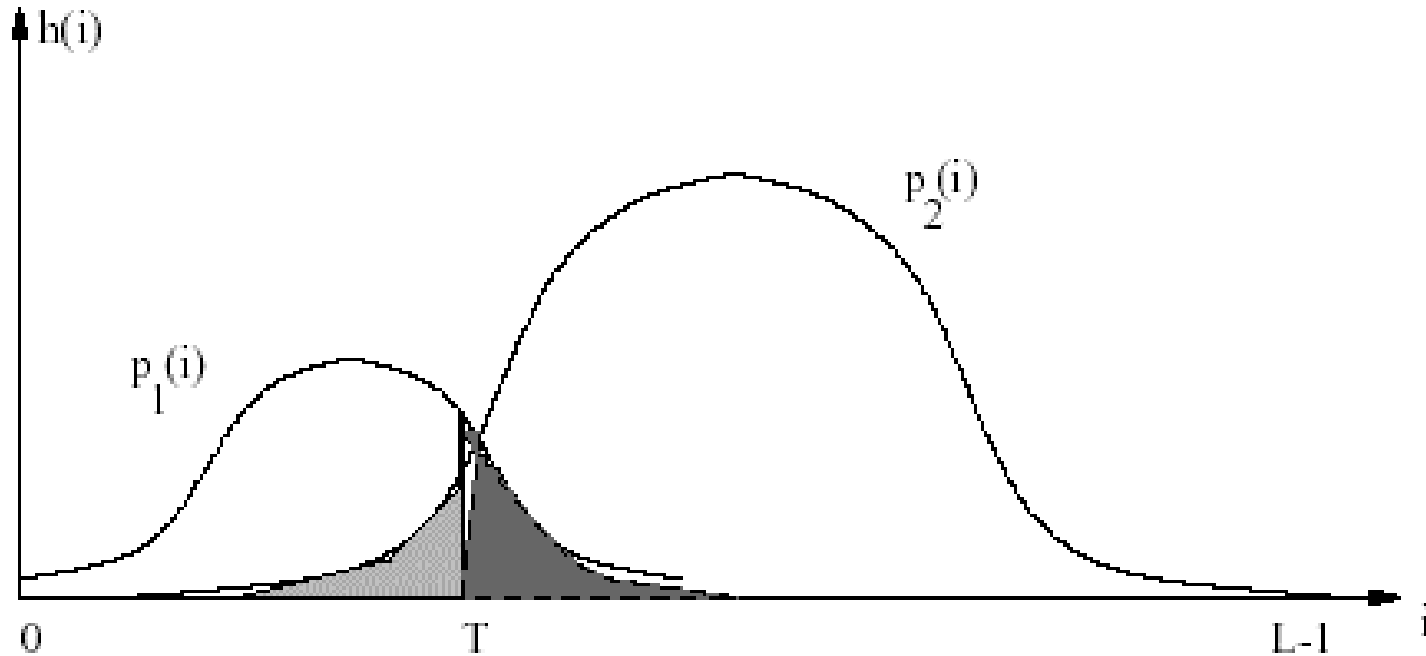
Pentru binarizare va trebui determinat pragul T ce separa modurile.

Pragul este “optim” in sensul minimizarii unei erori.

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Eroarea de segmentare este data de pixelii prost etichetati:
nivel de gri mai mic ca T , desi provin din p_2
nivel de gri mai mare ca T , desi provin din p_1



$$\varepsilon(T) = P_2 \int_{-\infty}^T p_2(x) dx + P_1 \int_T^{+\infty} p_1(x) dx$$

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Optim :

$$\frac{d\varepsilon(T)}{dT} = 0$$

$$\implies P_1 p_1(T) = P_2 p_2(T)$$

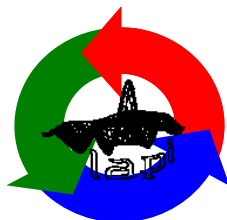
$$\varepsilon(T) = P_2 \int_{-\infty}^T p_2(x) dx + P_1 \int_T^{+\infty} p_1(x) dx$$

$$P_1 p_1(T) = P_2 p_2(T)$$

ecuatie rezolvabila (necunoscuta T), in conditiile in care se stiu probabilitatile claselor si forma functiilor de densitate de probabilitate ce modeleaza clasele.

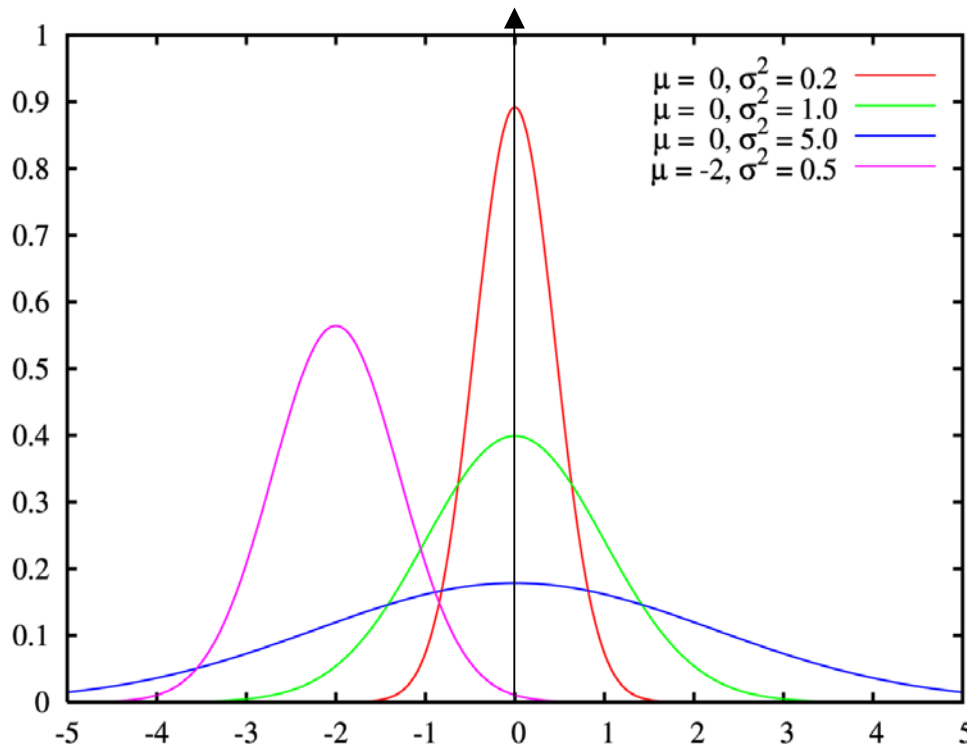
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In cazul particular cel mai curent, distributiile ce caracterizeaza obiectele sunt **normale (gaussiene)**.

$$N_i(\mu_i, \sigma_i)(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(x - \mu_i)^2}{2\sigma_i^2}\right)$$



Daca variantele claselor sunt egale, pragul este:

$$T = \frac{\mu_1 + \mu_2}{2} - \frac{\sigma^2}{\mu_1 - \mu_2} \ln \frac{P_1}{P_2}$$

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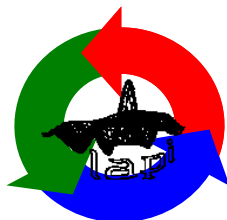


- costuri/ penalitati ?

- mai mult de doua clase ?

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SEGMENTAREA IMAGINILOR (ORIENTATA PE REGIUNI)

THRESHOLDING

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Segmentare globala

Metoda Bhattacharyya

Ipoteza: clasele de obiecte pot fi modelate cu distributii gaussiene.

Metoda: descompunerea histogramei imaginii in moduri normale.

Obs. ca :

$$N(\mu, \sigma)(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\ln N(\mu, \sigma)(x) = -\ln\sqrt{2\pi\sigma^2} - \frac{(x-\mu)^2}{2\sigma^2}$$

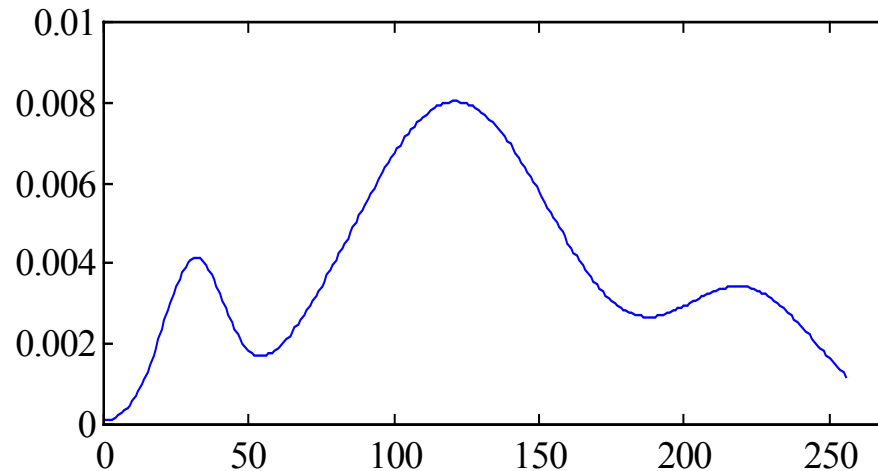
$$\frac{d}{dx} \ln N(\mu, \sigma)(x) = -\frac{x}{\sigma^2} + \frac{\mu}{\sigma^2} = mx + n$$

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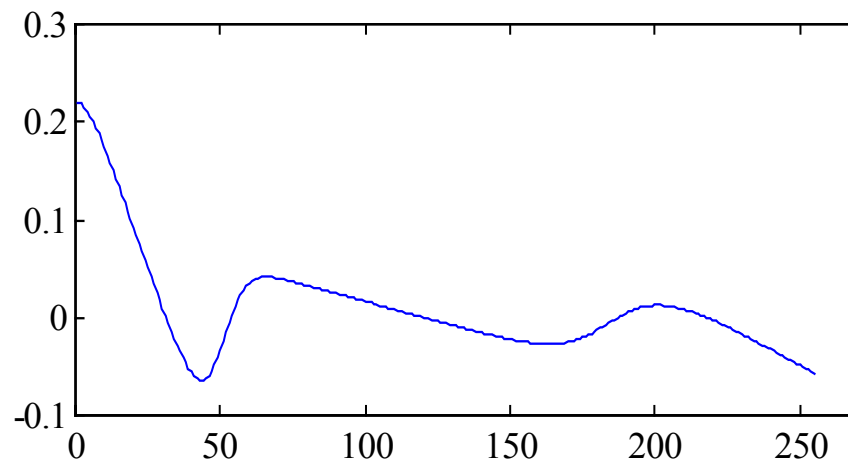


Unui mod gaussian ii corespunde o dreapta descrescatoare in domeniul functiei discriminant (derivata logaritmului densitatii de probabilitate).

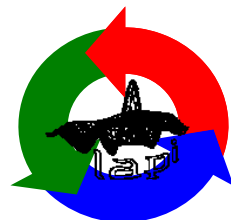
histograma
trimodala

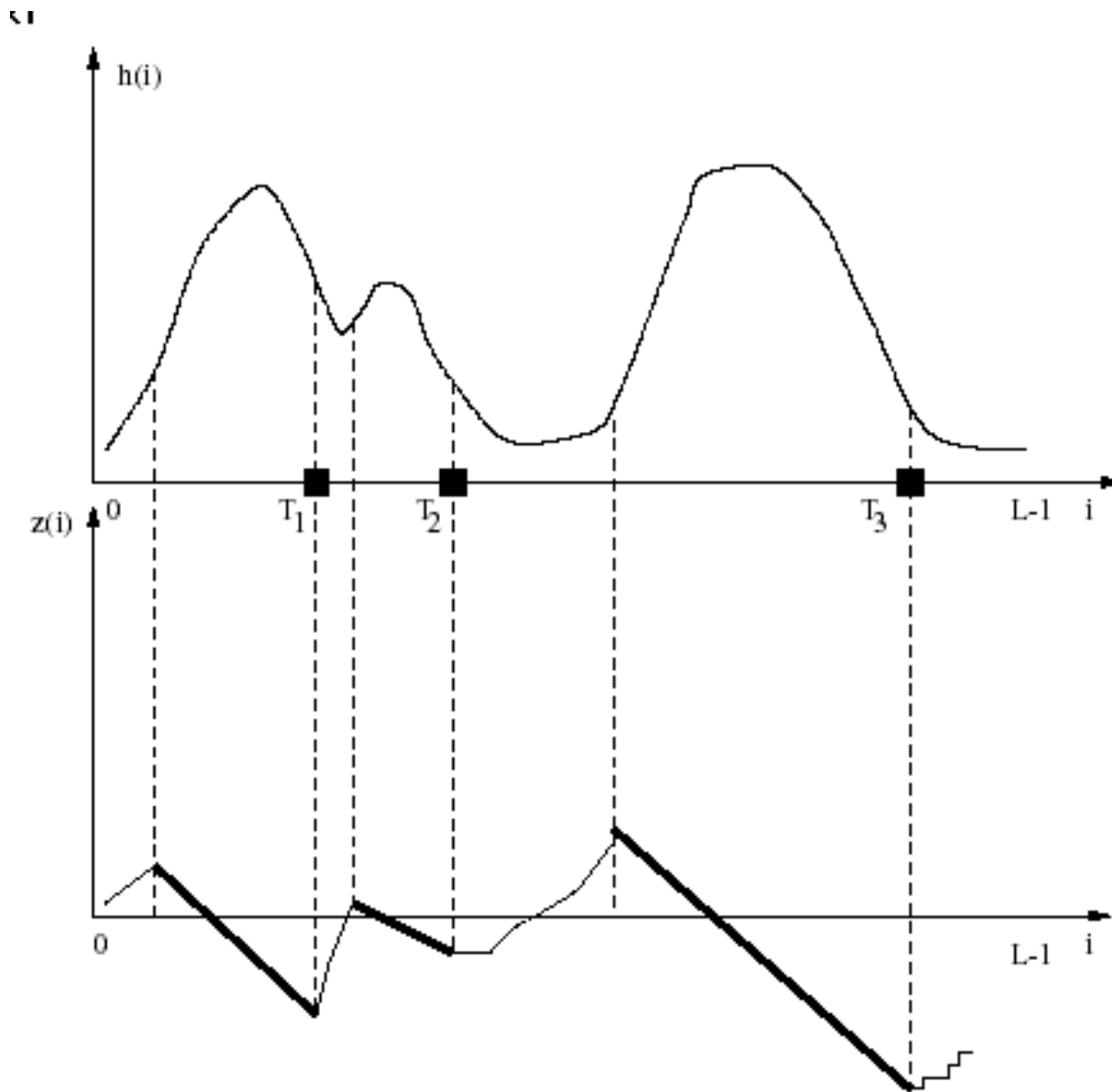


functia discriminant
(derivata logaritmului
histogramei)

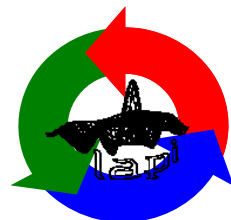


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In functia discriminant se vor identifica deci domeniile pe care functia este descrescatoare. Aceste domenii separa modurile normale in histograma.

Parametrii modurilor sunt obtinuti din parametrii drepte de aproximare a functiei discriminant.

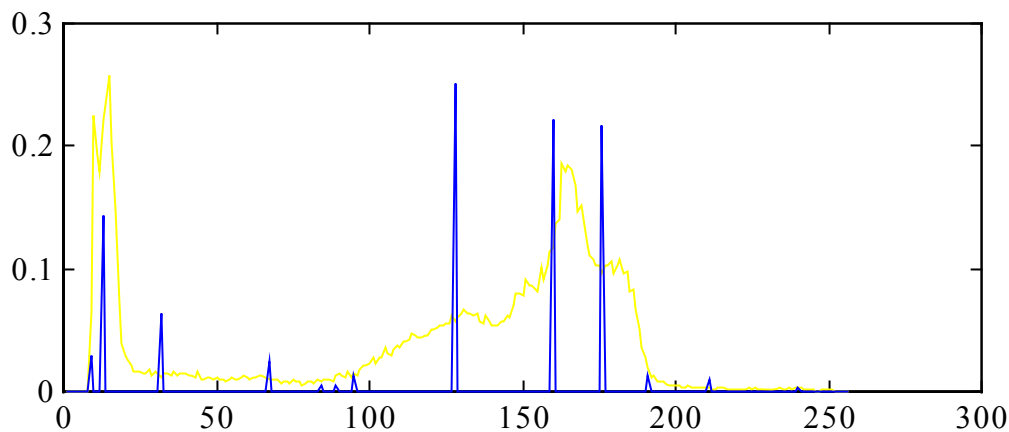
$$z(x) = \frac{d}{dx} \ln N(\mu, \sigma)(x) = -\frac{x}{\sigma^2} + \frac{\mu}{\sigma^2} = mx + n$$

Segmentarea se va face dupa praguri alese pe capetele intervalelor de descrestere a functiei discriminant.

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Alternativ: nivelele de gri din interiorul fiecarui mod sunt inlocuite cu media modului respectiv.



Segmentarea poate fi vazuta astfel ca o problema de aproximare a valorilor imaginii.

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LABORATORUL DE ANALIZA ȘI PRELUCRAREA IMAGINILOR





Original:
230 nivele gri



Aproximare 1:
22 nivele gri
SNR= 29.6 dB



Aproximare 2:
13 nivele gri
SNR=20.4 dB

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Original:
247 nivele gri

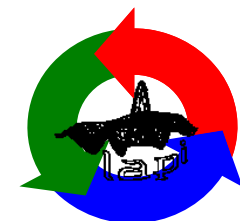


Aproximare 1:
40 nivele gri
SNR= 35 dB



Aproximare 2:
13 nivele gri
SNR=24 dB

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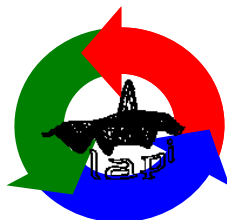
Observatii:

Numarul de moduri nu poate fi controlat decat prin impunerea unei lungimi minime a unui interval de descrestere a functiei discriminant si a unei erori mici de aproximare cu o dreapta.

Putem considera ca metoda imi asigura o legatura intre conceptele de segmentare si cuantizare (aproximare a valorilor initiale cu alte valori, in numar mai mic).

Apar probleme daca ipoteza de normalitate a modurilor nu este adevarata.

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Separabilitatea modurilor si masuri de eroare

Inegalitatea Cebasev:

Pentru orice variabila aleatoare de dispersie σ , probabilitatea ca variabila aleatoare sa fie mai departata cu $k \times \sigma$ fata de media sa este cel mult

$$\text{Prob}(|x - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

“aproape toate valorile sunt apropiate de medie”

Fata de o distributie normala, estimarea de mai sus este “larga”.

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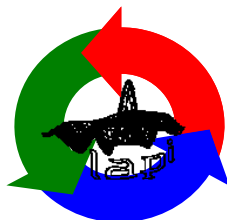
Inegalitatea Cantelli:

Derivata din inegalitatea anterioara (Cebasev) numai pentru $k > 0$ si valori mai mari ca media variabila aleatoare:

$$\text{Prob}(|x - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$



$$\text{Prob}(x - \mu \geq k\sigma) \leq \frac{1}{1 + k^2}$$



Inegalitatea Visochanski-Petunin:

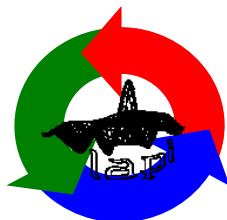
Derivata din inegalitatea anterioara (Cebasev) numai pentru distributii unimodale:

$$\text{Prob}(|x - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$



$$\text{Prob}(|x - \mu| \geq k\sigma) \leq \frac{4}{9k^2}, k \geq \sqrt{\frac{8}{3}}$$

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Inegalitatea Gauss:

Pentru orice variabila aleatoare de dispersie σ , probabilitatea ca variabila aleatoare sa fie mai departata cu $k \times \sigma$ fata de modul sau este cel mult

$$\text{Prob}(|x - m| \geq k\sigma) \leq \begin{cases} \frac{4}{9k^2} \cdot \frac{(\mu - m)^2 + \sigma^2}{\sigma^2}, & k \geq \sqrt{\frac{4}{3} \cdot \frac{(\mu - m)^2 + \sigma^2}{\sigma^2}} \\ 1 - k \sqrt{\frac{1}{3} \cdot \frac{(\mu - m)^2 + \sigma^2}{\sigma^2}}, & k < \sqrt{\frac{4}{3} \cdot \frac{(\mu - m)^2 + \sigma^2}{\sigma^2}} \end{cases}$$

$$\text{P}(|X - m| \geq k\tau) \leq \begin{cases} \left(\frac{2}{3k}\right)^2, & k \geq \frac{2}{\sqrt{3}} \\ 1 - \frac{k}{\sqrt{3}}, & 0 \leq k < \frac{2}{\sqrt{3}} \end{cases} \quad \tau^2 = \overline{(x - m)^2}$$

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Algoritmul EXPECTATION MAXIMIZATION (EM)

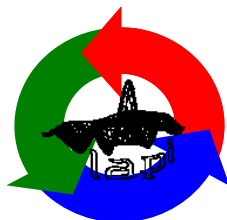
Se presupune ca valorile pixelilor sunt rezultatul unui amestec de distributii normale (GMM = Gaussian Mixture Models)

$$h(x) = \sum_{i=1}^K \alpha_i N_i(\mu_i, \sigma_i^2)(x) = f(x | \theta)$$

$$\theta = \{\alpha_i, \mu_i, \sigma_i^2\}_{i=1 \dots K}$$

$$\sum_{i=1}^K \alpha_i = 1$$

Cunoastem valorile individuale x si histograma $h(x)$, dorim sa gasim vectorul de parametri θ .



Obiectiv: dandu-se N observatii x , sa se gaseasca θ ca estimat de plauzibilitate maxima:

$$\theta = \arg \max_{\theta} f(x_1, \dots, x_N | \theta)$$

Algoritm:

1. Initializare

2. Pas E (expectation)

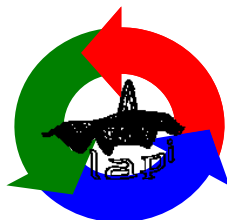
- Pe baza lui θ curent, fiecare punct de date este asociat unei gaussiene (ca probabilitate de apartenenta)

3. Pas M (maximization)

- Pe baza valorilor datelor, se determina noii parametri ai gaussianelor

4. Se repeta 2-3 pana la convergenta (\sim zeci de iteratii)

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Implementare EM

- Pas E (estimarea probabilitatii punctului t asociat distributiei gaussiene j):

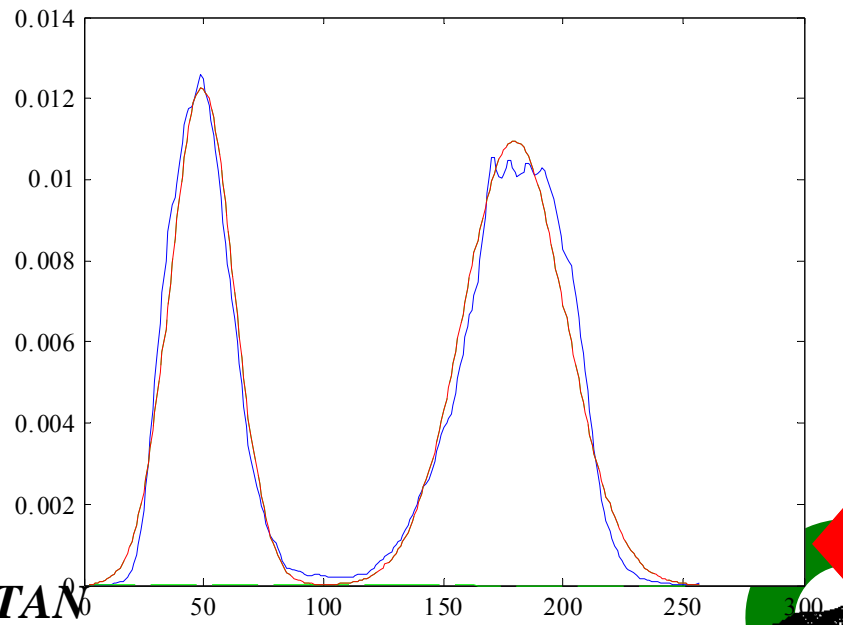
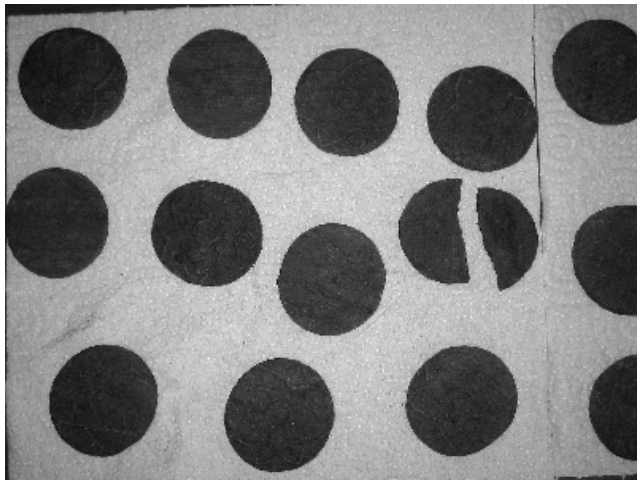
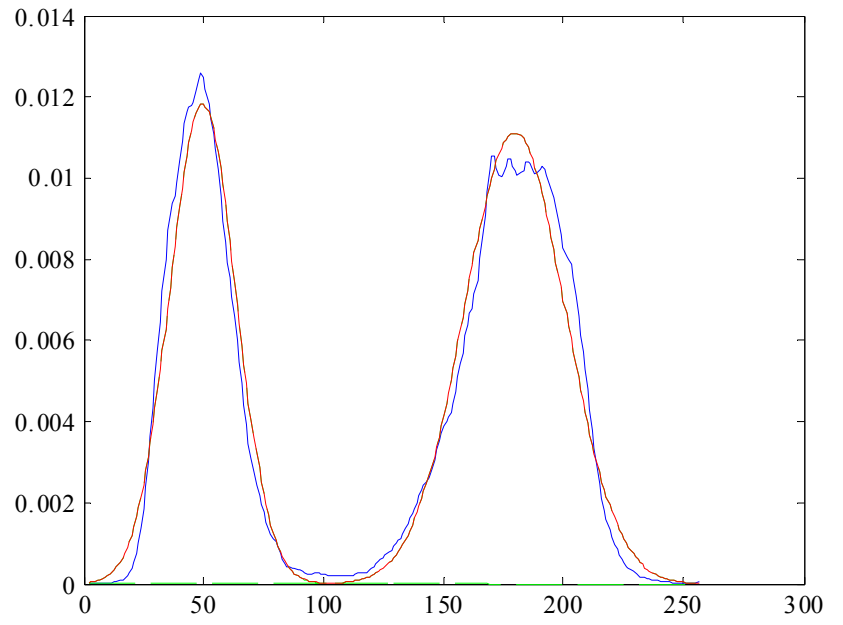
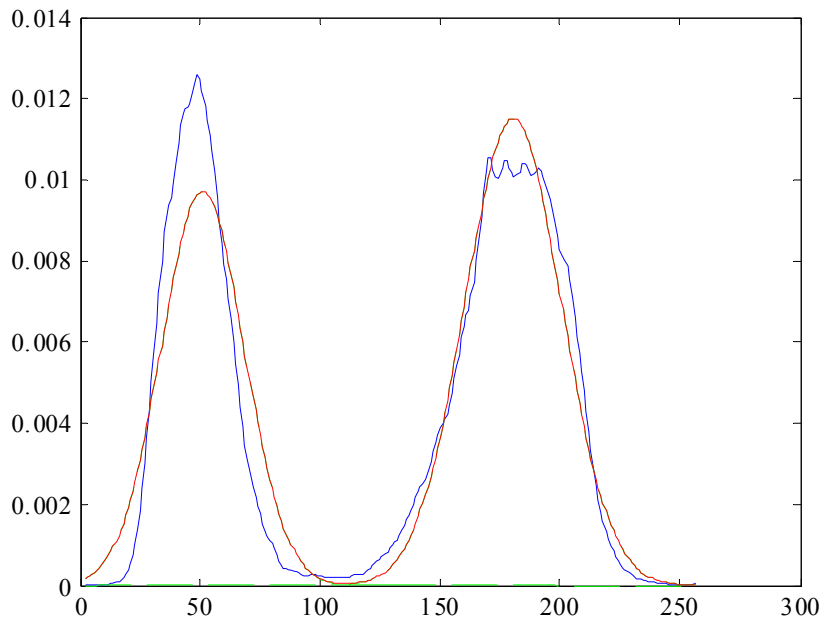
$$w_{t,j} = \frac{\alpha_j f(x_t | \mu_j, \Sigma_j)}{\sum_{i=1}^K \alpha_i f(x_t | \mu_i, \Sigma_i)} \quad j = 1, \dots, K \quad t = 1, \dots, N$$

- Pas M (estimare noi parametri):

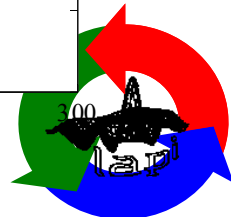
$$\alpha_j^{new} = \frac{1}{N} \sum_{t=1}^N w_{t,j}$$

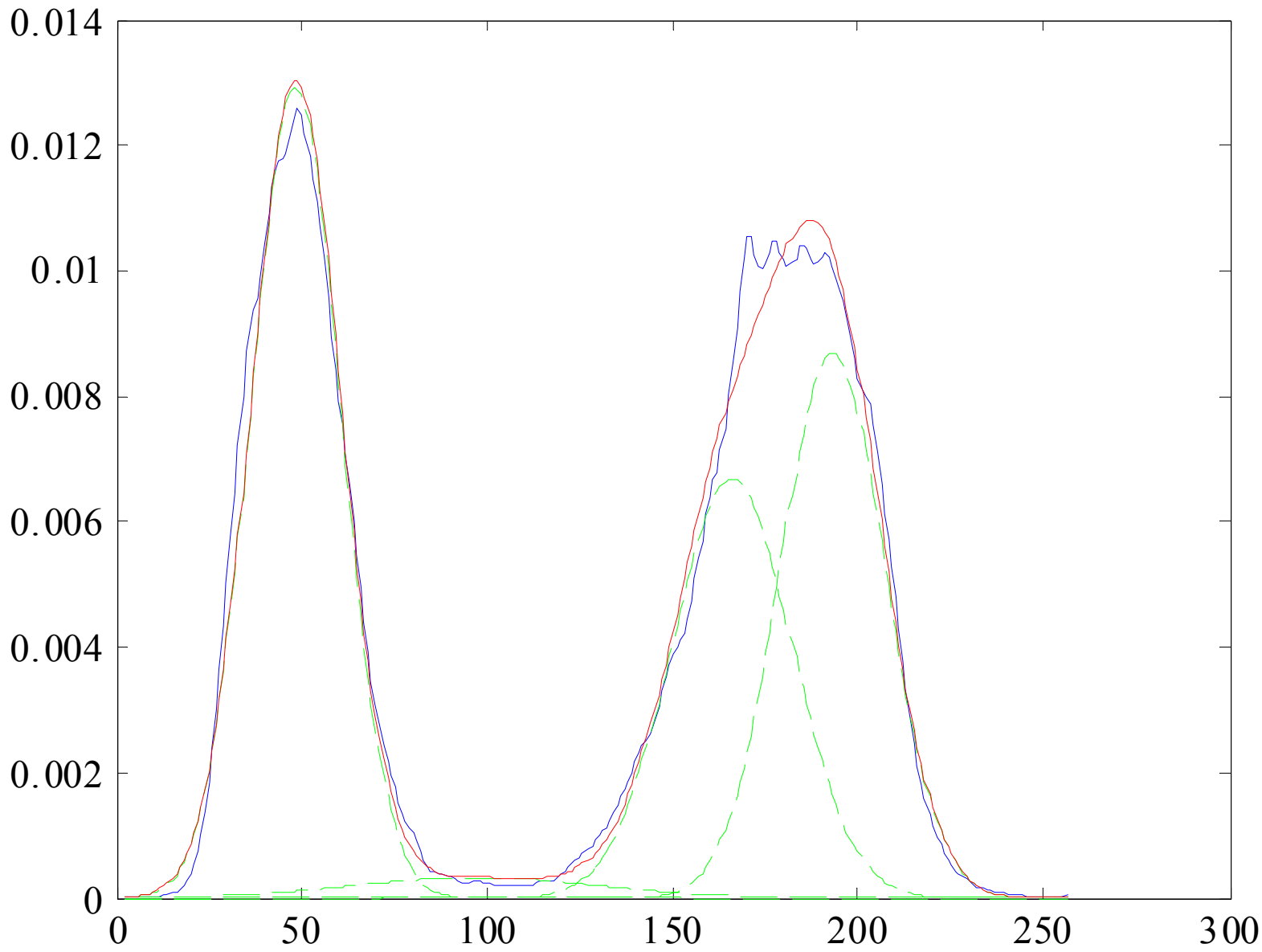
$$\mu_j^{new} = \frac{\sum_{t=1}^N w_{t,j} x_t}{\sum_{t=1}^N w_{t,j}}$$

$$\Sigma_j^{new} = \frac{\sum_{t=1}^N w_{t,j} (x_t - \mu_j^{new})(x_t - \mu_j^{new})^T}{\sum_{t=1}^N w_{t,j}}$$



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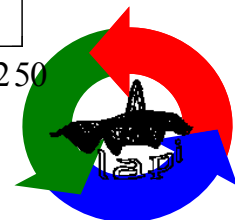
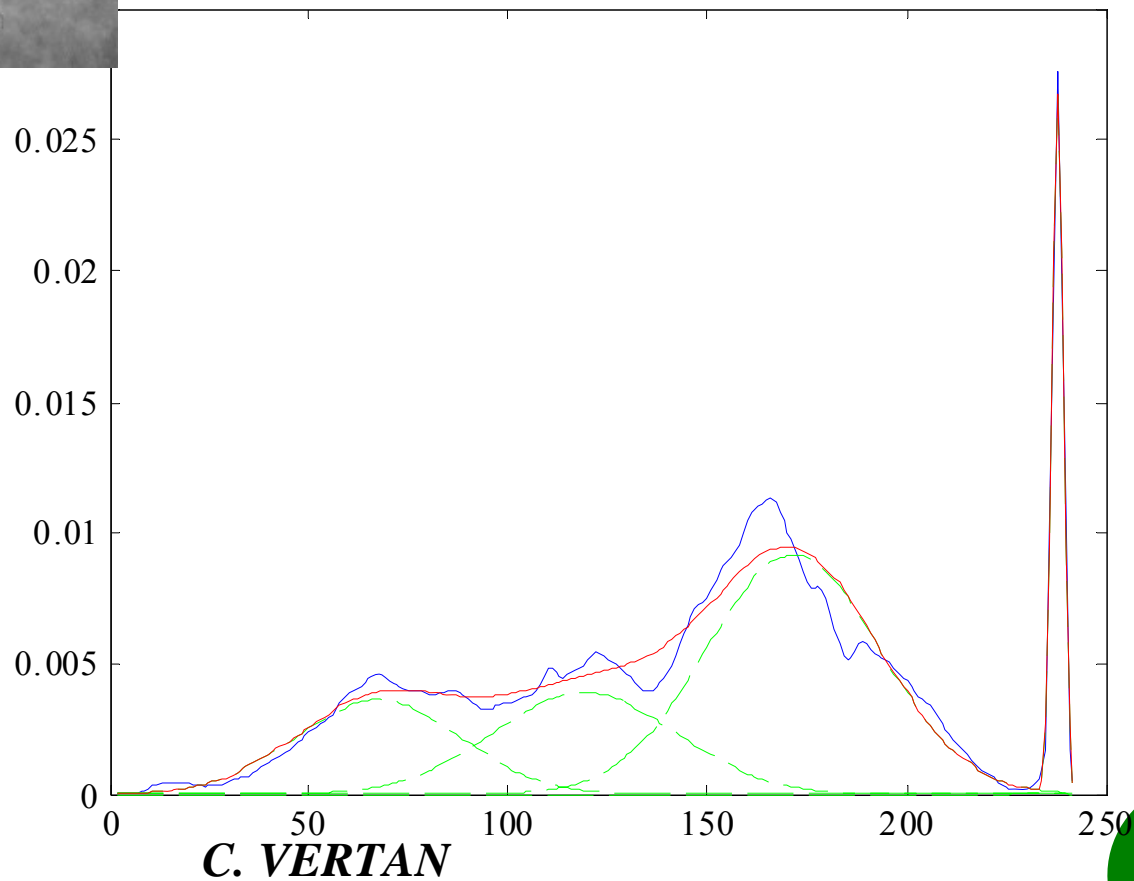


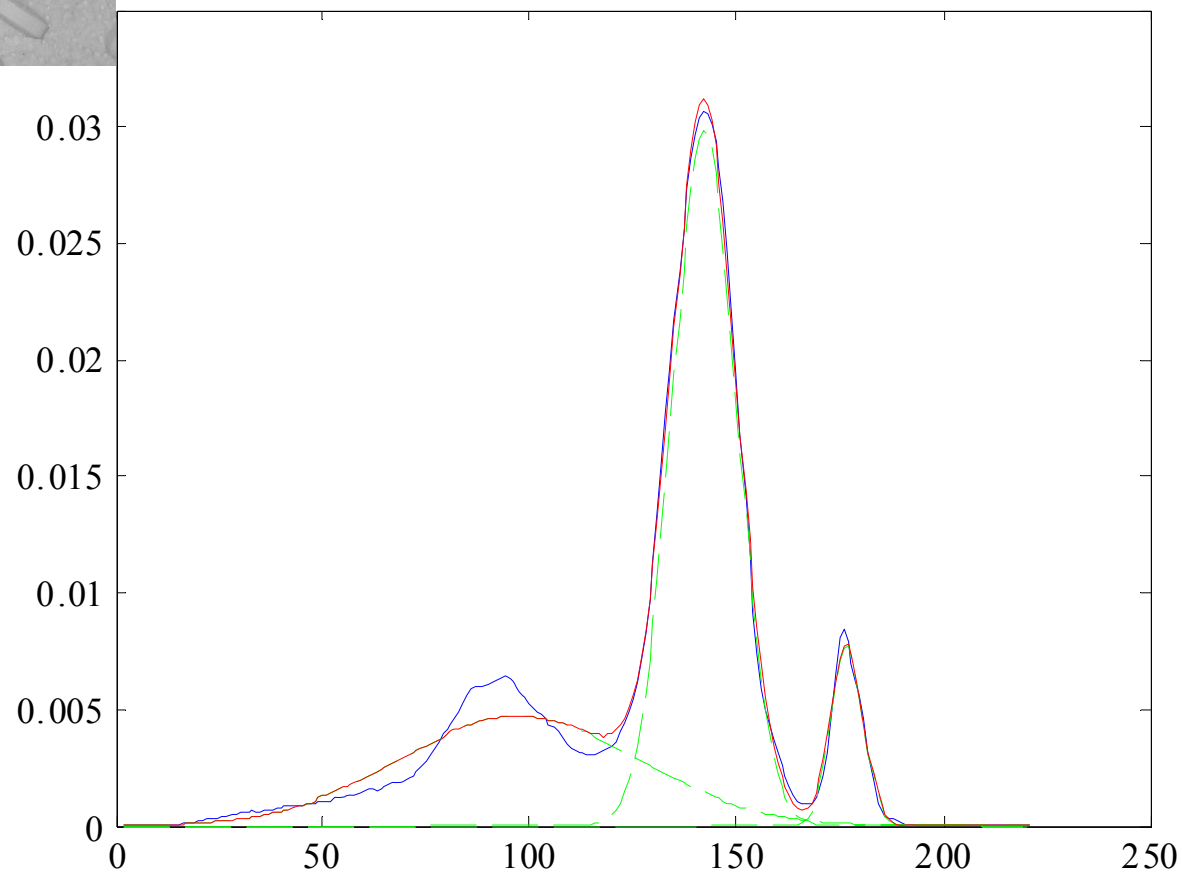


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LABORATORUL DE ANALIZA ȘI PRELUCRAREA IMAGINILOR

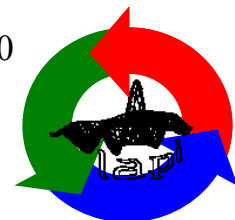






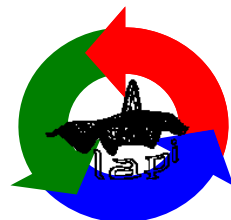
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Next: metode locale si adaptive.

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Segmentare globala

Metoda Kittler-Illingworth

Dar daca parametrii mixturii de distributii nu se cunosc ?

Metoda K-I propune un mod de estimare a parametrilor statistici ai modurilor histogramei prin functii criteriu legate de masura informatiei.

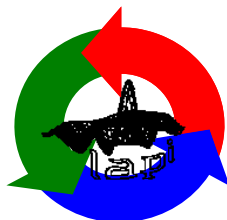
Cazul cel mai simplu: binarizarea.

Histograma reala : $h(i)$

Histograma bimodala teoretica: $p(i)$

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$$p(\tilde{i}) = P_1 N(\mu_1, \sigma_1)(\tilde{i}) + P_2 N(\mu_2, \sigma_2)(\tilde{i}) \longrightarrow h(i)$$

H - ipoteza ca nivelul i provine din h

H' - ipoteza ca nivelul i provine din p

$$\Pr(\tilde{i}|H) = h(\tilde{i}) \text{ și } \Pr(\tilde{i}|H') = p(\tilde{i}).$$

$$\Pr(H) = \Pr(H')$$

$$\frac{\Pr(H|\tilde{i})}{\Pr(H'|\tilde{i})} = \frac{\Pr(\tilde{i}|H) \Pr(H)}{\Pr(\tilde{i}|H') \Pr(H')} = \frac{h(\tilde{i}) \Pr(H)}{p(\tilde{i}) \Pr(H')}$$

$$\log \frac{h(\tilde{i})}{p(\tilde{i})} = \log \frac{\Pr(H|\tilde{i})}{\Pr(H'|\tilde{i})} - \log \frac{\Pr(H)}{\Pr(H')}.$$

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Informatia medie in favoarea uneia dintre ipotezele H_0/H_1 este data de:

$$J = \sum_{i=0}^{L-1} h(i) \log \frac{h(i)}{p(i)} = \sum_{i=0}^{L-1} h(i) \log h(i) - \sum_{i=0}^{L-1} h(i) \log p(i)$$

J de minimizat sau termenul 2 de maximizat

Cum am presupus ca mixtura este normala (gaussiana) si avem expresia pentru $p(i)$,

$$\bar{J} = \sum_{i=0}^T h(i) \log P_1 N(\mu_1, \sigma_1)(i) + \sum_{i=T+1}^{L-1} h(i) \log P_2 N(\mu_2, \sigma_2)(i)$$

$$\bar{J} = \log P_1 \sum_{i=0}^T h(i) + \log \frac{1}{\sqrt{2\pi\sigma_1^2}} \sum_{i=0}^T h(i) - \sum_{i=0}^T h(i) \frac{(i - \mu_1)^2}{\sigma_1^2} + \log P_2 \sum_{i=T+1}^{L-1} h(i) +$$

$$\log \frac{1}{\sqrt{2\pi\sigma_2^2}} \sum_{i=T+1}^{L-1} h(i) - \sum_{i=T+1}^{L-1} h(i) \frac{(i - \mu_2)^2}{\sigma_2^2}$$

$$\bar{J} = P_1 \log P_1 - \frac{1}{2} P_1 \log \sigma_1^2 - P_1 \log \sqrt{2\pi} - \frac{1}{2\sigma_1^2} \sum_{i=0}^T h(i) i^2 + \frac{1}{2\sigma_1^2} \sum_{i=0}^T h(i) i \mu_1 - \frac{1}{2\sigma_1^2} \sum_{i=0}^T h(i) \mu_1^2 +$$

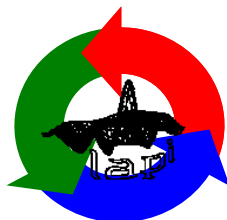
$$P_2 \log P_2 - \frac{1}{2} P_2 \log \sigma_2^2 - P_2 \log \sqrt{2\pi} - \frac{1}{2\sigma_2^2} \sum_{i=T+1}^{L-1} h(i) i^2 + \frac{1}{2\sigma_2^2} \sum_{i=T+1}^{L-1} h(i) i \mu_2 - \frac{1}{2\sigma_2^2} \sum_{i=T+1}^{L-1} h(i) \mu_2^2.$$

$$\bar{J} = P_1 \log P_1 + P_2 \log P_2 - \frac{1}{2} (P_1 \log \sigma_1^2 + P_2 \log \sigma_2^2) - \log \sqrt{2\pi} - \frac{1}{2\sigma_1^2} (P_1 \mu_1^2 - 2P_1 \mu_1 \bar{\mu}_1 + P_1 \bar{\mu}_1^2 + P_1 \bar{\sigma}_1^2) - \frac{1}{2\sigma_2^2} (P_2 \mu_2^2 - 2P_2 \mu_2 \bar{\mu}_2 + P_2 \bar{\mu}_2^2 + P_2 \bar{\sigma}_2^2)$$

Pentru o aproximare buna a parametrilor statistici ai modurilor

$$\bar{J} = P_1 \log P_1 + P_2 \log P_2 - \frac{1}{2} (P_1 \log \sigma_1^2 + P_2 \log \sigma_2^2) - \frac{1}{2} - \log \sqrt{2\pi}$$

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Segmentare globala

Metode entropice (Pun/ Kapur & Sahoo)

Trebuie maximizata informatia medie pe nivel de gri ce corespunde partitionarii cu un set de praguri de segmentare date.

$$T_{\text{opt}} = \arg \max [H_f(T) + H_b(T)] \quad \text{cu}$$
$$H_f(T) = - \sum_{g=0}^T \frac{p(g)}{P(T)} \log \frac{p(g)}{P(T)} \quad \text{and} \quad H_b(T) = - \sum_{g=T+1}^G \frac{p(g)}{P(T)} \log \frac{p(g)}{P(T)}$$
$$P(g) = \sum_{i=0}^g p(i).$$

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Separabilitatea Nakagawa

Pentru moduri vecine normale, $N_1(\mu_1, \sigma_1)(x)$ și $N_2(\mu_2, \sigma_2)(x)$
separabilitatea modurilor histogramei inseamna:

$$\left\{ \begin{array}{l} \mu_2 - \mu_1 > 4 \\ 0.1 < \frac{\sigma_2}{\sigma_1} < 10 \\ \min_{x \in [\mu_1; \mu_2]} h(x) < 0.8 \cdot \min(h(\mu_1), h(\mu_2)) \end{array} \right.$$

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